Supplementary material for the paper

Motivating a supplier to test product quality

by
Yaron Yehezkel[1]

January 2013

1. Additional characteristics of the equilibrium contract

In this section I specify the characteristics of the equilibrium contract for motivating a supplier to test product quality in more details. As it is possible to think of a wide set of distribution functions that satisfy the condition that the vertical integration payoff is higher in state H than in state L, the distortion in the equilibrium quantity, which is a direct result of the sign of $F_L(\theta) - F_H(\theta)$, may vary substantially. Moreover, as I will show below, the distortion is not directly related to the mean and variance of the two distributions. I will therefore offer some polar cases of distribution functions, and then move to characterizing the solution even further by making specific assumptions on the demand and cost.

The first and most natural polar case to think of is the case where $F_H(\theta)$ dominates $F_L(\theta)$ by FOSD: $F_L(\theta) > F_H(\theta)$, $\forall\theta\in(\theta_0, \theta_1)$. Since $V(q^*(\theta);\theta) - c(q^*(\theta))$ is increasing with θ, FOSD always satisfies the assumption $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$. Applying Proposition 1 yields:

Corollary 1: *Suppose that $F_H(\theta)$ dominates $F_L(\theta)$ by FOSD. Then, in the interior solution to the buyer's problem, the equilibrium quantity is distorted downwards for* $\forall\theta\in(\theta_0, \theta_1)$, *and equals the vertical integration quantity at the two extremes of the support,* θ_0 *and* θ_1. *Moreover, the equilibrium payment to the supplier is increasing with* θ *for* $\forall\theta\in[\theta_0, \theta_1]$.

The case of FOSD is illustrated in Panel (a) of Figure 1. For the case of FOSD, the quantity is distorted downwards regardless of the average. As for the equilibrium payment to the supplier, $F_L(\theta) > F_H(\theta)$ implies that $T^{**}(\theta) - c(q^{**}(\theta))$ is increasing with θ and since $c(q^{**}(\theta))$ is increasing with θ, $T_\theta^{**}(\theta) > 0$. Notice that $T^{**}(\theta)$ can therefore be negative for

[1] The Leon Recanati Graduate School of Business Administration, Tel Aviv University, Ramat Aviv, Tel Aviv, 69978, Israel. Email: <yehezkel@post.tau.ac.il>.

low realizations of θ. I elaborate on the implications of this result for public policy in the next section.

The second polar scenario concerns cases where the two distribution functions cannot be ranked according to FOSD. This implies that there is at least one interior intersection point between $F_H(\theta)$ and $F_L(\theta)$. To generate clean and intuitive predictions concerning the quantity distortion in this case, I follow Diamond and Stiglitz (1974) by assuming that $F_H(\theta)$ and $F_L(\theta)$ satisfy the *single-crossing condition*. Applying Proposition 1 yields:

Corollary 2: *Suppose that there is exactly one* $\theta^C \in (\theta_0, \theta_1)$ *such that* $F_H(\theta^C) = F_L(\theta^C)$. *Then, in the interior solution to the buyer's problem,*

(i) *if* $f_L(\theta^C) < f_H(\theta^C)$, *then the equilibrium quantity is distorted downwards (upwards) for* $\theta \in (\theta_0, \theta^C)$ $(\theta \in (\theta^C, \theta_1))$;

(ii) *if* $f_L(\theta^C) > f_H(\theta^C)$, *then the equilibrium quantity is distorted upwards (downwards) for* $\theta \in (\theta_0, \theta^C)$ $(\theta \in (\theta^C, \theta_1))$.

The case where $F_H(\theta)$ and $F_L(\theta)$ satisfy the *single-crossing condition* is illustrated in panels (b) and (c) of Figure 1. Panel (b) illustrates part (i) of Corollary 2 in which $F_H(\theta)$ intersects $F_L(\theta)$ from below, such that the quantity is distorted downwards for low values of θ and upwards for high values of θ. In this case a government, for example, will use a new defense system less than is socially desirable if product testing indicated that its quality is below expectations and more than is socially desirable otherwise. A downstream firm such as supermarket or an automobile manufacturer will price the new product above the monopoly price (which corresponds to buying a quantity below the monopoly quantity) for low realizations of quality and below the monopoly price otherwise. Panel (c) illustrates part (ii) where $F_H(\theta)$ intersects $F_L(\theta)$ from above and the quantity distortion is completely reversed. Now the results may seem somewhat counter-intuitive as a government, for example, will actually use a new defense system *more* than is socially desirable even though product testing indicated that its quality is low, and *less* than is socially desirable otherwise. Likewise, a supermarket or an automobile manufacturer will price the new product above the monopoly price for low realizations of quality and below the monopoly price otherwise. Notice however that if in addition to the single-crossing property, $F_H(\theta)$ dominates $F_L(\theta)$ by Second Order Stochastic Dominance (SOSD), then only the first scenario (part (i)) is possible.[2] As for the payment to the supplier, here the effect of θ on $T^{**}(\theta)$ is inconclusive because $T^{**}(\theta)$ –

[2] This is because SOSD requires that $\int_{\theta_0}^{\theta} F_L(\hat{\theta})d\hat{\theta} \geq \int_{\theta_0}^{\theta} F_H(\hat{\theta})d\hat{\theta}$, $\forall \theta \in [\theta_0, \theta_1]$.

$c(q^*(\theta))$ is decreasing with θ whenever $F_L(\theta) < F_H(\theta)$, while $c(q^*(\theta))$ is also increasing with θ.

The problem with the single-crossing condition, as well as with SOSD, is that these two features may not satisfy the assumption $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$. Intuitively, SOSD ensures $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$ only if $V(q^*(\theta);\theta) - c(q^*(\theta))$ is concave in θ. However,

$$\frac{d^2}{d\theta^2}\big(V(q^*(\theta);\theta) - c(q^*(\theta))\big) = V_{\theta\theta}(q^*(\theta);\theta) + V_{q\theta}(q^*(\theta);\theta)q_\theta^*(\theta)\,. \tag{1}$$

Since by assumption $V_{q\theta}(q;\theta) > 0$ and since $q_\theta^*(\theta) > 0$, (1) is positive if $V_{\theta\theta}(q^*(\theta);\theta)$ is either positive or negative but small in absolute terms. This raises the question of whether the two cases in Corollary 2 are possible under that assumption that $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$, and how these two cases are affected by the shape of the distribution functions and their variance and mean.

The answer to the above question depends on the specific shape of the two distribution functions and on the buyer's payoff. To show that indeed the two cases in Corollary 2 are feasible, and to characterize the solution even further, I turn to making more specific assumptions on the two distribution functions and the demand. Consider first the two distribution functions. Suppose that θ is distributed in state H along the unit interval according to some probability distribution function $f_H(\theta)$ with mean $\mu_H = 1/2$ and variance σ_H^2. In state L, $f_L(\theta)$ is a triangle transformation of $f_H(\theta)$ in that $f_L(\theta) = f_H(\theta) + g(\theta)$, where

$$g(\theta) = \begin{cases} \dfrac{(\beta-2\theta)(\alpha-1)}{\beta}, & if\ \theta\in[0,\beta], \\ \dfrac{(2\theta-\beta-1)(\alpha-1)}{1-\beta}, & if\ \theta\in[\beta,1], \end{cases} \tag{2}$$

and $0 < \alpha < 2$, $0 < \beta < 1$. Figure 2 illustrates the probability and cumulative distribution functions given $g(\theta)$. For illustrative reasons only the figure shows the case where $f_H(\theta)$ is the uniform distribution. The analysis below allows for any $f_H(\theta)$, including those that are nonlinear in θ, in which case $f_L(\theta)$ is also nonlinear. As the figure illustrates, the shape of the gap between $f_L(\theta)$ and $f_H(\theta)$ and between $F_L(\theta)$ and $F_H(\theta)$ is determined by the parameters α and β in the following way. The parameter α determines the gap in the weight that the two distributions place on the extremes of the support. If $0 < \alpha < 1$ ($1 < \alpha < 2$), then state L places less (more) weight on the extremes of the support than state H, as illustrated in Panel (a) (Panel (b)) of Figure 2. Consequently, $F_L(\theta)$ and $F_H(\theta)$ satisfy the single-crossing condition and if $0 < \alpha < 1$ ($1 < \alpha < 2$), $F_L(\theta^C)$ crosses $F_H(\theta^C)$ from below (above). The parameter β measures the skewness of the gap between the two probability distribution functions. If $\beta <$ ($>$) 1/2, the gap is skewed to the left (right) side of the support. Notice that for any $F_H(\theta)$, α and β, the two cumulative distribution functions intersect exactly at $\theta^C = \beta$. The two parameters α and β also determine the mean and variance in state L. Using the simplifying assumption that $\mu_H = ½$, I can express μ_L and σ_L^2 only as a function of α, β and σ_H^2 for any given $f_H(\theta)$ in the following way:

$$\mu_L = \left(2 + \alpha + 2\beta(1-\alpha)\right)\tfrac{1}{6}, \quad \sigma_L^2 = \sigma_H^2 + (1-\alpha)\left(\alpha(1-2\beta)^2 - 2\beta(1-\beta) - 1\right)\tfrac{1}{36}. \tag{3}$$

Next consider the demand function. Suppose that $p(q;\theta) = \theta - q$, and $c(q) = 0$. Therefore, $q^*(\theta) = \theta/2$ and $V(q^*(\theta);\theta) - c(q^*(\theta)) = \theta^2/4$. I can now explicitly write the equilibrium quantity, parameterized by the Lagrange multiplier, λ, as:

$$q^{**}(\theta) = \begin{cases} \dfrac{\theta}{2} - \lambda(\alpha-1)\left[\dfrac{(\beta-\theta)\theta}{2\left(\beta f_H(\theta) + \lambda(\alpha-1)(\beta-2\theta)\right)}\right], & if\ \theta \in [0,\beta], \\ \dfrac{\theta}{2} - \lambda(1-\alpha)\left[\dfrac{(\theta-\beta)(1-\theta)}{2\left(f_H(\theta)(1-\beta) + \lambda(1-\alpha)(1-\beta-2\theta)\right)}\right], & if\ \theta \in [\beta,1], \end{cases} \tag{4}$$

where the proof of Proposition 1 establishes that $\lambda > 0$ and is increasing with $C/\gamma(1-\gamma)$.[3] The first term in each line, $\theta/2$, is the full-information quantity. The term in the squared brackets in each line is positive for $\theta \in (\theta_0, \theta_1)$ and equal to zero at $\theta = \{\theta_0, \theta_1\}$ because the second-order condition requires that both denominators in the second terms are positive.[4] Finally, I can use (2) and (3) to specify the conditions on α, β, σ_H^2 , μ_L and σ_L^2 (where recall that by assumption $\mu_H = 1/2$) that satisfy the assumption that $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$.

Figure 3 provides a full characterization of the example's parameters and their effect on the quantity distortion. Consider first the effect of α and β. The assumption that $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$ allows for two possibilities. The first is the case where $0 < \alpha < 1$ and $0 < \beta < 0.618$ (the lower left-hand side box in Figure 3). Intuitively, under linear demand the buyer is a "risk lover" in the sense that the buyer's payoff is convex in θ. Consequently, given that $0 < \alpha < 1$ such that state H places more weight on the extremes of

[3] In this example $q^{**}(\theta)$ is not differentiable at $\theta = \beta$. However, IC_B still holds in this case because $q^{**}(\theta)$ is continuous and increasing with θ at $\theta = \beta$ as long as λ is sufficiently small.

[4] The second order condition is always satisfied if $f_H(\theta)$ is sufficiently high or λ is sufficiently low.

the support than state L, the buyer prefers state H over state L for all $\beta < 1/2 = \mu_H$ and also for $\beta > 1/2$ as long as β is not too high. In this case $F_L(\beta)$ intersects $F_H(\beta)$ from below and therefore, consistent with part (ii) of Corollary 2, (4) reveals that the contract admits upward (downward) distortion in the equilibrium quantity for $\theta \in [0, \beta]$, ($\theta \in [\beta, 1]$). The second possibility is the case where $1 < \alpha < 2$ and $0.618 < \beta < 1$ (the upper right-hand side box in Figure 3). Now, state H places less weight on the two extremes of the support than state L and the buyer prefers state H over state L only if β is sufficiently high. Consistent with part (i) of Corollary 2, (4) reveals that the contract admits downward (upward) distortion in the equilibrium quantity for $\theta \in [0, \beta]$, ($\theta \in [\beta, 1]$). In both possibilities, α and β fully characterize the direction of the quantity distortion. Notice that FOSD is a special case of this example, in which $\beta = 0$ and $0 < \alpha < 1$, or equivalently $\beta = 1$ and $1 < \alpha < 2$. Consistent with Corollary 1, (4) reveals that the quantity is distorted downwards for all $\theta \in (\theta_0, \theta_1)$.

Figure 3 also reveals that the effect of the mean and variance of the two distributions on the direction of the distortion is less conclusive than the effect of α and β. Notice that I can write $E_k(V(q^*(\theta);\theta) - c(q^*(\theta))) = E_k\theta^2/4 = ((E_k\theta)^2 + E_k(\theta - E_k\theta)^2)/4 = (\mu_k^2 + \sigma_k^2)/4$, $k = \{L,H\}$. It follows that with linear demand and no cost, $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$ only requires that $\mu_H^2 + \sigma_H^2 > \mu_L^2 + \sigma_L^2$. Intuitively, since $V(q^*(\theta);\theta) - c(q^*(\theta))$ is convex in θ, if $\mu_H = \mu_L$, the buyer prefers the state with the higher variance. Moreover, if $\sigma_H^2 = \sigma_L^2$, the buyer prefers the state with the higher mean. Figure 3 reveals that if both $\mu_H > \mu_L$ and $\sigma_H^2 > \sigma_L^2$, then the direction of the distortion is inconclusive, as $\mu_H > \mu_L$ and $\sigma_H^2 > \sigma_L^2$ can emerge both in the lower left-hand side box where there is first upward and then downward distortion in the equilibrium quantity, or in the upper right-hand side box in which the quantity distortion is completely reversed. Intuitively, $\mu_H > \mu_L$ and $\sigma_H^2 > \sigma_L^2$ can emerge when state H places higher weight on the extremes of the support than state L and the gap is skewed to left, or when state H places lower weight on the extremes of the support than state L and the gap is significantly skewed to right. If however both $\mu_H > \mu_L$ and $\sigma_H^2 < \sigma_L^2$ then the quantity is distorted first downwards and then upwards (the upper right-hand side box). Alternatively, if both $\mu_H < \mu_L$ and $\sigma_H^2 > \sigma_L^2$ then the quantity is distorted first upwards and then downwards (the lower left-hand side box). The model does not allow for the last possibility of both $\mu_H < \mu_L$ and $\sigma_H^2 < \sigma_L^2$, as the assumption that state H is the preferable one requires that $\mu_H^2 + \sigma_H^2 > \mu_L^2 + \sigma_L^2$. Finally, notice that the case where state L is a mean persevering spread of state H, $\mu_H = \mu_L$, falls on the lower left-hand side box where quantity is distorted first upwards and then downwards.

To conclude, the main point of the above analysis is that the direction of the quantity distortion is directly affected by the gap between $F_L(\theta)$ and $F_H(\theta)$. Even though I considered only a single-crossing between $F_L(\theta)$ and $F_H(\theta)$, the results can be extended to any number of intersection points, θ_1^C, …, θ_n^C that satisfy $F_L(\theta_i^C) = F_H(\theta_i^C)$, $i = \{1,\ldots,n\}$. In this case if $f_L(\theta_i^C)$ < (>) $f_H(\theta_i^C)$, then for θ slightly below θ_i^C, there is downward (upward) distortion in the quantity while for θ slightly above θ_i^C there is upward (downward) distortion in the quantity. Finally, the same argument can also apply for the case where instead of intersection points, there are intervals of θ in which $F_L(\theta) = F_H(\theta)$.

2. The robustness of Proposition 1 to more than 2 states

This section discusses the robustness of Proposition 1 to the case of more than two states. I show that when the buyer is limited to offering a menu that can only discriminate among θ (but not among states), then along the lines of my paper, quantity distortion should be affected by the gap between the weighted average of all the cumulative distributions in which the buyer deals with the supplier, and the weighted average of all the cumulative distributions in which the buyer does not deal with the supplier.

Suppose that there are $N > 2$ states. In each state, $k \in \{1, \ldots, N\}$, θ is drawn from a distribution function $f_k(\theta)$, with a cumulative distribution function $F_k(\theta)$. Each state occurs with probability γ_k, where $0 < \gamma_k < 1$ and

$$\sum_{k=1}^{N} \gamma_k = 1 .$$

If the buyer wants to deal with the supplier in all states, then as in my paper, the buyer will not ask the supplier to test its new product. Suppose instead that the buyer wants to deal with the supplier only in "high" states. In particular, suppose that $E_k(V(q^*(\theta);\theta) - c(q^*(\theta)))$ is increasing with k, and that

$$E_1(V(q^*(\theta);\theta) - c(q^*(\theta))) < V^* < E_N(V(q^*(\theta);\theta) - c(q^*(\theta))).$$

This assumption is consistent with condition (2) in my paper, and it implies that the buyer wants to deal with the supplier only for high states.

As in my model, the supplier can perform a test at a cost C, that enables the supplier to learn the state, k, but not θ. The buyer learns θ ex-post, if the buyer agrees to use the new product instead of the old one. Quality information is unverifiable.

Suppose that the buyer constructs a contract, $(T(\theta),q(\theta))$, that motivates the supplier to test the new product, and then to accept the contract only in states $k \geq k^C$. Notice that the buyer can endogenously choose k^C. The buyer then sells the old product and earns V^* for all states $k \in [1, k^C - 1]$, and sells the new product an earns $E_k(V(q(\theta);\theta) - c(q(\theta)))$ otherwise. The buyer's problem is:

$$\max_{(q(\theta),T(\theta),k^C)} \sum_{k=1}^{k^C-1} \gamma_k V^* + \sum_{k=k^C}^{N} \gamma_k E_k\left(V(q(\theta);\theta)-T(\theta)\right),$$

s.t.

$$(IC_B) \qquad \forall\theta\in[\theta_0,\theta_1], \quad \theta = \arg\max_{\tilde\theta}\left(V(q(\tilde\theta);\theta)-T(\tilde\theta)\right).$$

$$(IC_S^{ex-post}) \qquad E_k\big(T(\theta)-c(q(\theta))\big)\le 0, \quad \forall k\in\{1,\dots,k^C-1\},$$

$$(IR_S^{ex-post}) \qquad E_k\big(T(\theta)-c(q(\theta))\big)\ge 0, \quad \forall k\in\{k^C,\dots,N\},$$

$$(IC_S^{ex-ante}) \quad -C+\sum_{k=k^C}^{N}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right)\ge \sum_{k=1}^{N}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right),$$

$$(IR_S^{ex-ante}) \quad -C+\sum_{k=k^C}^{N}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right)\ge 0.$$

Notice that this problem is qualitatively similar to the maximization problem (5) in the paper. The only difference is that here there are $k^C - 1$ $IC_S^{ex-post}$ constraints, because the buyer needs to ensure that the supplier will not accept the contract for all realization of $k < k^C$. Likewise, there are N - k^C $IR_S^{ex-post}$ constraints, because the buyer needs to ensure that the supplier will accept the contract for all realizations of $k \ge k^C$. As in the binary type case, the first-best (and full information) contract, ($q(\theta)$, $T(\theta)$) = ($q^*(\theta)$, $c(q^*(\theta))$), satisfies all the ex-post constraints in equality, but does not satisfy the two ex-ante constraints. However, unlike the binary case, even if the two ex-post constraints are satisfied, they do not necessarily satisfy the ex-ante constraints.

To simplify these constraints, consider first $IC_S^{ex-ante}$. Notice that:

$$\sum_{k=k^C}^{N}\left[\gamma_k E_k\left(T(\theta)-c(q(\theta))\right)\right] = \sum_{k=k^C}^{N}\left[\gamma_k\int_{\theta_0}^{\theta_1}\left(T(\theta)-c(q(\theta))\right)f_k(\theta)d\theta\right] = \hat\gamma\int_{\theta_0}^{\theta_1}\left[\left(T(\theta)-c(q(\theta))\right)\sum_{k=k^C}^{N}\frac{\gamma_k}{\hat\gamma}f_k(\theta)\right]d\theta$$

$$=\hat\gamma\int_{\theta_0}^{\theta_1}\left[\left(T(\theta)-c(q(\theta))\right)\hat f_H(\theta)\right]d\theta,$$

where

$$\hat f_H(\theta)\equiv\sum_{k=k^C}^{N}\frac{\gamma_k}{\hat\gamma}f_k(\theta) \quad , \quad \hat\gamma\equiv\sum_{k=k^C}^{N}\gamma_k .$$

Then, I can define in a similar way $\hat f_L(\theta)$, and rewrite the ex-ante incentive compatibility constraint as:

$$-C+\sum_{k=k^C}^{N}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right)\geq\sum_{k=1}^{k^C-1}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right)+\sum_{k=k^C}^{N}\gamma_k E_k\left(T(\theta)-c(q(\theta))\right),$$

or:

$$-C+\hat{\gamma}\hat{E}_H\left(T(\theta)-c(q(\theta))\right)\geq(1-\hat{\gamma})\hat{E}_L\left(T(\theta)-c(q(\theta))\right)+\hat{\gamma}\hat{E}_H\left(T(\theta)-c(q(\theta))\right),$$

where $\hat{E}_H$ denote the expectation given $\hat{f}_H(\theta)$ (and similarly for $\hat{E}_L$). It is possible to see that the condition above is similar to the $IC_S^{ex-ante}$ constraint in the binary state case with the exception that now each distribution is the weighted average of all distributions in which the buyer uses and does not use the new product. The same argument holds for the supplier's ex-ante individual rationality constraint and the buyer's expected profit. Therefore, constraint (9) and Lemma 1 also follows through (with the new definitions of $\hat{E}_H$ and $\hat{E}_L$). This implies that for any given k^C such that $1 < k^C < N$, the resulting quantity distortion is a function of the gap between the two weighted averages of the cumulative distribution functions:

$$\hat{F}_H(\theta)\equiv\sum_{k=k^C}^{N}\frac{\gamma_k}{\hat{\gamma}}F_k(\theta),\qquad\hat{F}_L(\theta)\equiv\sum_{k=1}^{k^C-1}\frac{\gamma_k}{\hat{\gamma}}F_k(\theta).$$

The intuition comes from the restriction that the contract is the same for all states, and therefore it is possible to take both the supplier's and the buyer's payoff as common divisor.

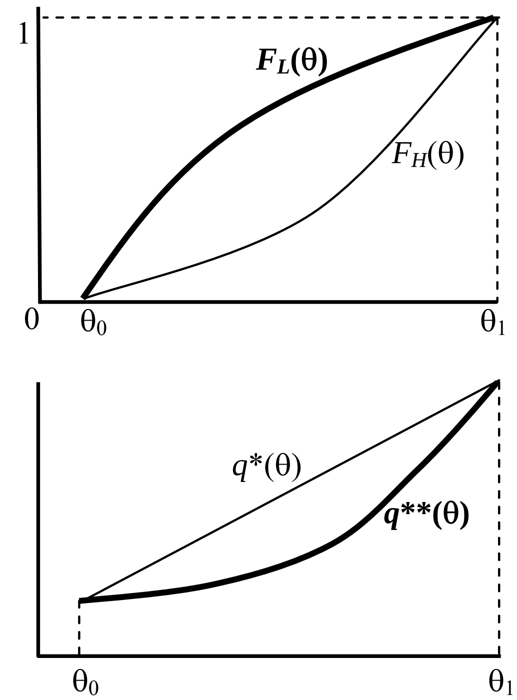
Remarks:

- In the case of $N > 2$, the buyer can also endogenously choose k^C. It is possible to solve the above maximization problem as a two-stage process. In the first stage, the buyer solves for the optimal menu given any k^C. In the second stage, the buyer selects the optimal k^C. The analysis above holds given $1 < k^C < N$, and therefore holds regardless of the k^C that the buyer chooses. Notice that the buyer may choose a different k^C than in the full information case, depending on the degree of the quantity distortion.
- In the case of $N > 2$ states, there are also the supplier's ex-post incentive compatibility constraints to consider: $E_k\left(T(\theta)-c(q(\theta))\right)\leq 0$ for $\forall k\in\{1,...,k^C-1\}$ and $E_k\left(T(\theta)-c(q(\theta))\right)\geq 0$ for $\forall k\in\{k^C,...,N\}$. In the binary case they never bind, but I cannot make the same argument for $N > 2$ without considering specific cases and using numerical simulations. At the same time, as (9) binds, quantity distortion is still going to be affected by the gap between the cumulative distribution functions.

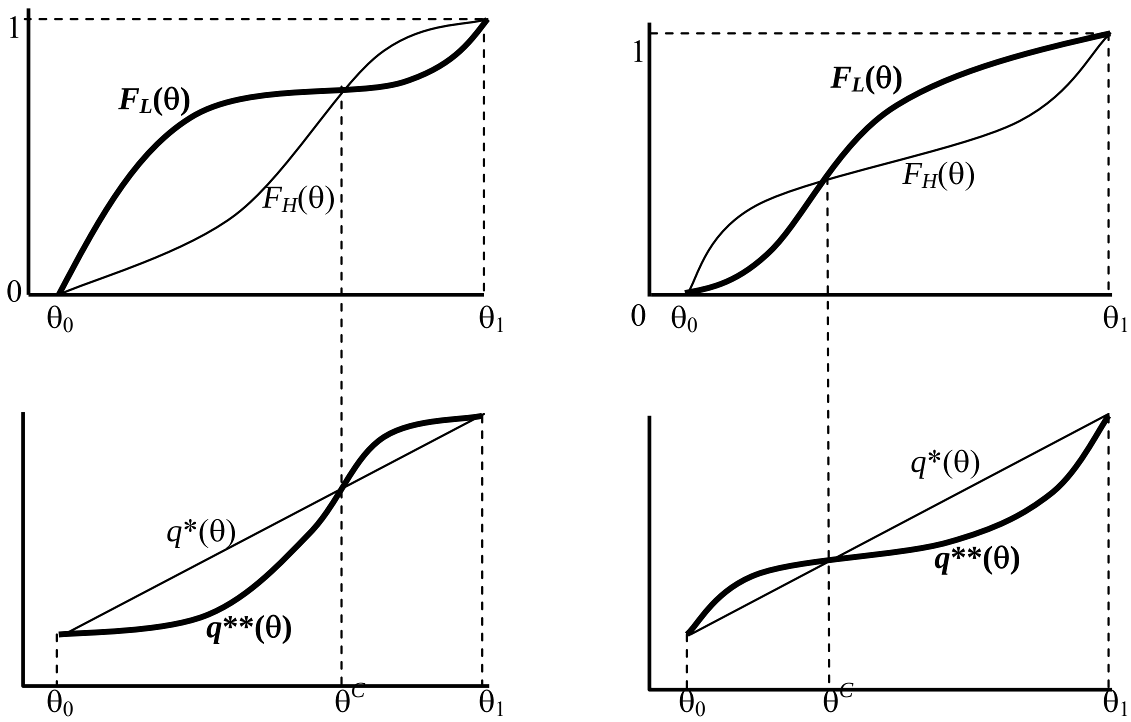
- The analysis above will not follow to the case where the buyer can offer a menu that discriminates between states: ($T_k(\theta)$, $q_k(\theta)$), such that the supplier chooses a different menu depending on the actual k. In such a case, the buyer also needs to motivate the supplier to choose the menu that corresponds to the true state. This adds a new set of constraints that insure that the supplier will report the true state. Given the true state, k, and the supplier's report, $\tilde{k}$, the menu should ensure that:

$$E_k\big(T_k(\theta) - c(q_k(\theta))\big) \geq E_k\big(T_{\tilde{k}}(\theta) - c(q_{\tilde{k}}(\theta)\big), \quad \forall k,\ \tilde{k} \in \{k^C, ..., N\} .$$

- These cosntraints may requier the buyer to pay the supplier ex-post information rents, which in turn may create another incentive for quantity distortion. As explained in the paper, I leave this point for future research.



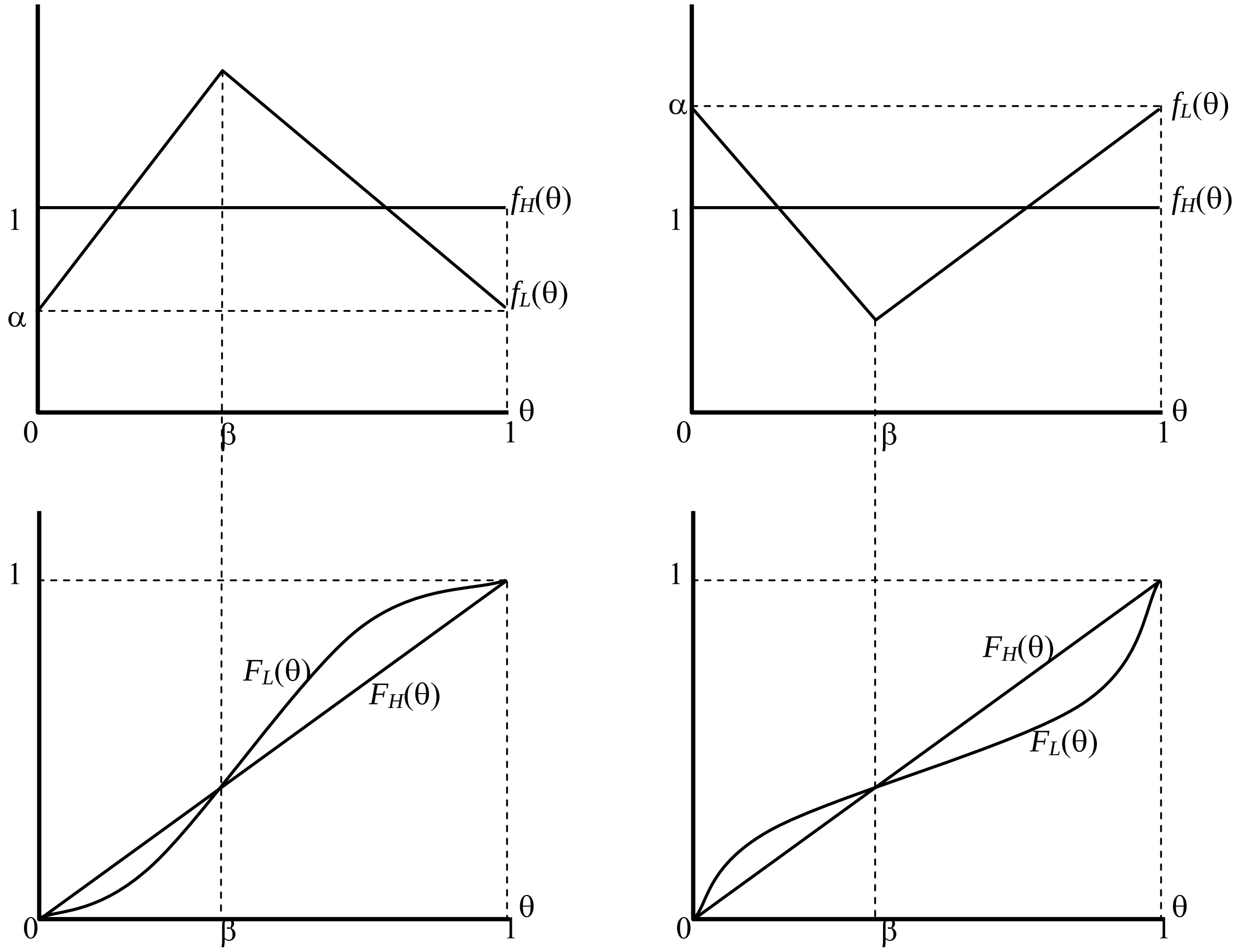
Panel (a): $F_L(\theta) > F_H(\theta) \ \ \forall\theta\in(\theta_0, \theta_1)$



Panel (b): $F_L(\theta) > F_H(\theta) \ \ \forall\theta\in(\theta_0, \theta^C)$,
$F_L(\theta) < F_H(\theta) \ \ \forall\theta\in(\theta^C, \theta_1)$

Panel (c): $F_L(\theta) < F_H(\theta) \ \ \forall\theta\in(\theta_0, \theta^C)$,
$F_L(\theta) > F_H(\theta) \ \ \forall\theta\in(\theta^C, \theta_1)$

Figure 1: The optimal mechanism under asymmetric information



Panel (a): $0 < \alpha < 1$

Panel (b): $1 < \alpha < 2$

Figure 2: The distribution functions given $g(\theta)$ (when $f_H(\theta)$ is the uniform distribution)

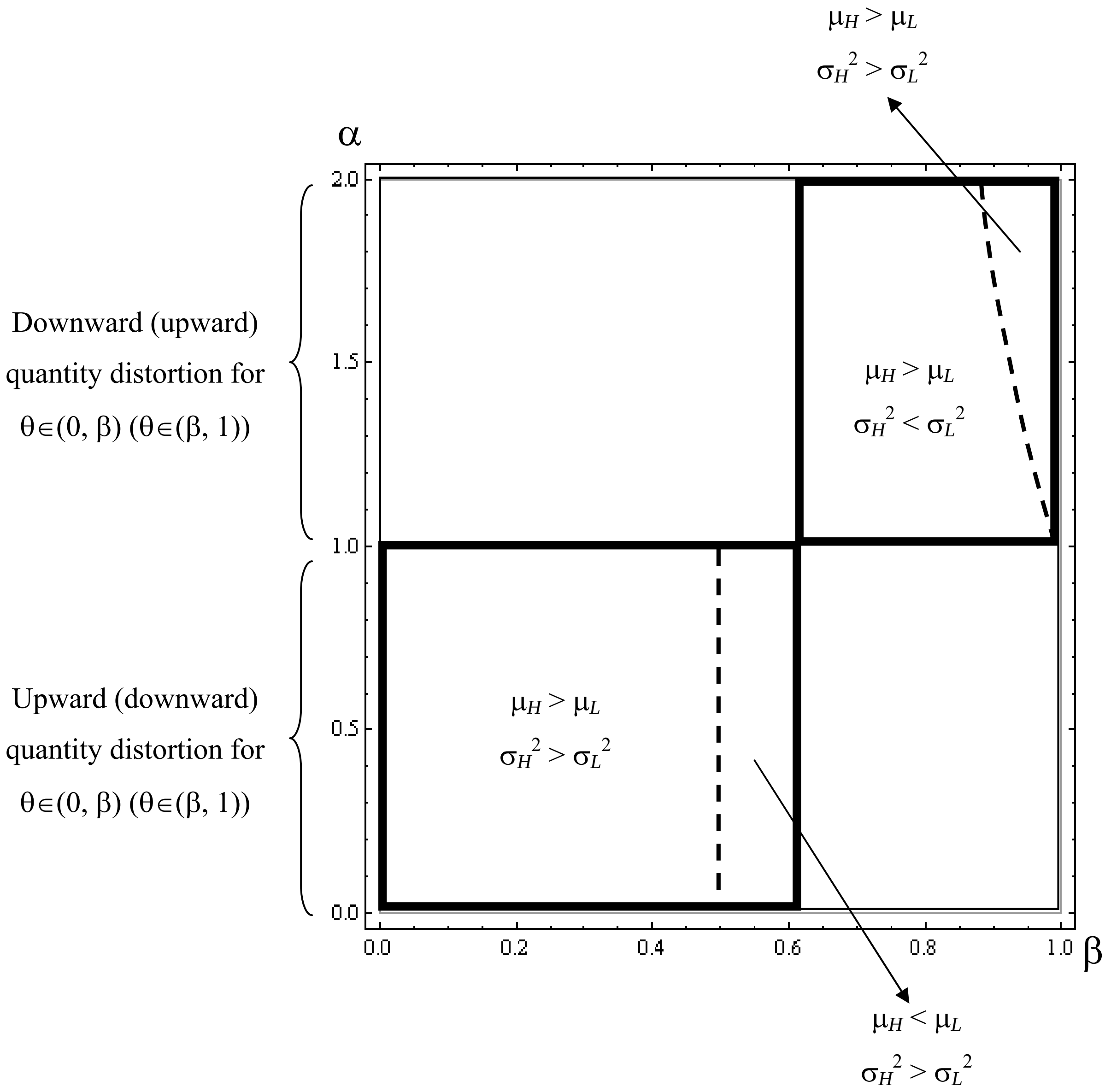


Figure 3: The effect of α, β, μ_H, μ_L, σ_H^2 and σ_L^2 on the direction of the quantity distortion

For $\mu_H = 1/2$. The areas in the bold line satisfy the assumption $E_H(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_L(V(q^*(\theta);\theta) - c(q^*(\theta)))$.