Signaling quality in an oligopoly when some consumers are informed

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***Abstract*:** This paper considers a signaling game between two competing firms and consumers. The firms have common private information concerning their qualities, and some of the consumers are informed about the firms' qualities. Firms use prices and uninformative advertising as signals of quality. The model reveals that in the separating equilibrium prices are first climbing and then declining with the proportion of informed consumers, while the expenditure on uninformative advertising is declining. Firms' profits are highest when the proportion of informed consumers is at an intermediate level. Pooling equilibria exist if the proportion of informed consumers is below a certain threshold.

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1. Introduction

In the process of introducing experience goods, firms may face consumers who are informed about the quality of their goods, while others are uninformed. It is possible to think of several explanations for this asymmetry in consumers' information. First, some consumers may have special knowledge or expertise that enables them to evaluate the quality of new brands better than others. For example, a consumer with a background in mechanics may be better able to asses the quality of a new car than other consumers. Second, consumer magazines usually publish reviews on new brands and potential consumers who read these magazines are better informed about the qualities of new brands than those who do not. Third, some consumers may be exposed to informative advertising that directly reveals the characteristics of the product, while others are not.

Thus, firms can take advantage of the presence of informed consumers to signal the qualities of their products to the uninformed. Such signals may include uninformative advertising that, unlike informative advertising, does not directly reveal the specifications of the product and serves only as a way to 'burn' money as a signal of the firm's confidence in its brand. Another signal of quality can be the price, or a combination of price and uninformative advertising.

This paper addresses the question of the combination of price and uninformative advertising that firms use to signal quality, in markets in which some consumers are informed, and how these signals, as well as the firms' profits and consumer welfare, vary with the proportion of informed consumers. I believe that this question is interesting for two reasons. First, it is reasonable to expect that as the market evolves along time, information will diffuse and more consumers will become informed. The question of how prices, advertising and profits vary with the proportion of informed consumers may also explain how they evolve along time, when they are used as a means for signaling quality. Second, instead of signaling quality through prices and uninformative advertising, firms may invest in informative advertising that directly increases the proportion of informed consumers. Therefore, the answer to this question also explains why firms may choose not to use informative advertising, and prefer instead to signal their qualities by other means, such as prices, uninformative advertising, or both.

This paper considers a signaling game between two competing firms and consumers. The firms have common private information concerning their qualities, and some of the consumers are informed about the firms' qualities. The product of one of the firm's is higher in quality than the other (firms *H* and firm *L*, respectively). Uninformed consumers know that one of the firms produces a superior brand, but, unlike informed consumers, they cannot distinguish which one it is. The two firms know their qualities, and use prices and uninformative advertising as signals.

The crucial feature of this setting is that uninformed consumers observe *two* sets of signals, from the two competing firms, for the *same* state. Firms therefore compete on consumers' beliefs: each firm wants consumers to believe that its product is of high quality while that of its competitor is of low quality.

The paper establishes two main results. First, the separating prices, as well as the profits of the two firms, are an inverted U-shaped function of the proportion of informed consumers. In particular, they are lower (higher) than their full information levels when the proportion of informed consumers is low (medium), and are identical to the full information levels when the proportion of informed consumers is high. Second, firm *H* invests in uninformative advertising whenever the proportion of informed consumers is low. Moreover, the expenditure on uninformative advertising is decreasing with the proportion of informed consumers, and for an intermediate proportion of informed consumers, firm *H* signals quality using only the price.

Intuitively, it is reasonable to expect that firm *H* will always signal quality by setting a high price, because doing so will deter firm *L* from mimicking it, as it will lose the sales of the informed consumers, who will not be willing to pay a high price for a low quality product. This argument however does not apply to the case of competition. Under competition, if firm *H* sets a high price to signal quality, then the competing firm *L* may have a stronger incentive to mimic *H* by setting the same price because doing so has the benefit of obtaining collusion at high prices between the two firms in their competition on the uninformed consumers. This effect is stronger the more consumers are uninformed because mimicking *H* only softens competition on the sales to the uninformed. If the proportion of informed consumers is low, this effect is strong such that firm *H* distorts its price downwards in comparison with the full information equilibrium in order to deter firm *L* from mimicking. As the proportion of informed consumers increases, the collusive effect of mimicking *H* becomes weaker, and the separating price of *H* increases above the full information price. For a sufficiently high proportion of informed consumers, firm *L* finds it less attractive to mimic *H* because doing so can only mislead a few consumers, and the separating price of firm *H* decreases until it reaches the full information price.

As for the expenditure on uninformative advertising, if the proportion of informed consumers is low, then price alone is insufficient for signaling quality, and firm *H* needs to use the additional tool of uninformative advertising. As the proportion of informed consumers increases, the price becomes a more effective tool for signaling quality, and firm *H* can use it to reduce the expenditure needed for separation; thus the expenditure on uninformative advertising is decreasing with the proportion of informed consumers.

In markets in which information diffuses over time, these results indicate that under competition, prices and profits climb in the introductory stage of the market and only then decline until they reach their full information levels. Moreover, these results can explain why firms may choose not to invest in informative advertising. The model show that the firms' profits are the highest for intermediate levels of informed consumers, implying that firms are better off leaving some consumers uninformed and signaling to these consumers through prices, uninformative advertising, or both.

The model also admits pooling equilibria. However, all pooling equilibria fail to satisfy a competitive version of the intuitive criterion whenever the proportion of informed consumers exceeds a certain threshold. Thus pooling equilibria are only possible at the initial stage of the market, and prices will still have to be climbing whenever the proportion of informed consumers reaches some threshold.

Most of the previous literature on signaling quality has focused on the monopoly case. Milgrom and Roberts (1986) and Bagwell (1987) show that when consumers make repeat purchases, a monopoly distorts its price downwards to signal high quality (or low costs as in Bagwell (1987)). Bagwell and Riorden (1991) consider a similar setting to this paper, in which a monopoly signals quality to consumers when some consumers are informed. Intuitively, in the monopoly case, if firm *L* mimics a high separating price of firm *H*, it does not benefit from collusion in competing on the uninformed consumers, as in the competitive case considered here. Therefore, they find that the monopoly signals quality by distorting its price upwards, and the separating price always decreases with the proportion of informed consumers. This paper contributes to the Bagwell and Riorden (1991) analysis by showing that the result of high and declining prices as signals for quality does not apply to the case of competition. Judd and Riordan (1994) show that a monopoly distorts its price upwards to signal high quality even without cost asymmetries between qualities. In their model, consumer utility is composed of a population-average component and an individual-specific component. A monopolist can signal that the population-average component is high through a high price because consumers already have some supporting information. This paper follows a similar intuition in that here the firms' signals coincide with what some consumers already know, but it also shows that prices can be distorted upwards or downwards even though costs are the same for both types. Linnemer (2002) considers a monopoly that can use both prices and uninformative advertising to signal quality, and shows that the monopoly uses advertising when the proportion of informed consumers is intermediate, though the price is always high and declining with the proportion of informed consumers.

In the context of price competition, Wolinsky (1983) considers a model in which competing firms can signal quality through prices, and consumers who can observe the quality of a firm before purchase at a cost. Wolinsky finds that firms distort their prices upwards to signal quality, even though firms compete. This difference in results emerges because Wolinsky assumes that each firm can observe only its own quality, whereas I consider common private information in which the two firms know each other's type. In a closely related paper, Hertzendorf and Overgaard (2001) consider two vertically differentiated firms with negatively correlated qualities, and show that the high quality firm signals quality through a high (low) price when the degree of vertical differentiation is high (low). In this paper I follow the same equilibrium concept as in Hertzendorf and Overgaard, by focusing on firms that are ex-ante symmetric and considering an equilibrium refinement which is based on the assumption that a unilateral deviation from a separating equilibrium does not change consumers' beliefs. Fluet and Garella (2002) consider a signaling game in a duopoly in which the qualities of the two firms are not perfectly correlated, and derive conditions for the existence of separating equilibria with and without advertising. Hertzendorf and Overgaard (2002) also consider a duopoly with non-correlated qualities, and focus on the case where the firms cannot use uninformative advertising. They show that even though separating equilibria exist, they fail to satisfy an equilibrium refinement based on minimality. Bontems and Meunier (forthcoming) consider the same equilibrium concept as in Hertzendorf and Overgaard (2001) (and in this paper) in the context of a duopoly when both firms can choose their levels of horizontal differentiation. They show that asymmetric information increases the possibility of maximum horizontal differentiation. Daughety and Reinganum (2007) consider a signaling game in a duopoly in which each firm has private information concerning its own type and find that in a separating equilibrium prices are distorted upwards. Moreover, the separating prices are increasing functions of the prior probability of high quality. Barigozzi, Garella and Peitz (2007) consider comparative advertising, in which an entrant makes a costly claim that its quality is as good as that of an incumbent and the incumbent can contest this claim in court. Vanin (2007) considers a signaling game with endogenous entry and quality selection, and finds that average quality may decrease with the degree of substitution between firms.

This paper contributes to the above literature by considering the possibility that some consumers are informed and by showing how prices, uninformative advertising and profits vary with the proportion of informed consumers.

The rest of the paper is organized as follows. The next section presents the model and full information benchmark. Section 3 defines the subgame perfect Bayesian equilibria. Sections 4 and 5 solve the separating and pooling equilibria, respectively. Section 6 concludes and discusses the robustness of the results. All proofs are in Appendix A.

2. The model

Consider two firms, a high quality firm (H for short) and a low quality firm (L for short). The two firms compete by setting prices simultaneously. Suppose that the production costs of the two firms are zero. I focus on identical production costs for two reasons. First, as Judd and Riordan (1994) point out, when quality is measured not in terms of objective characteristics but rather in terms of the utility the brand provides to consumers (which is the case in this model), then such *perceived* quality may not necessarily be more costly to produce. Second, as I explain in the concluding section, the assumption of identical cost does not qualitatively affect the results, though it makes the analysis more tractable.

Turing to the demand side, suppose that the total mass of consumers is 1. Following Bagwell and Riorden (1991), each consumer buys at most one unit of the brand, and is willing to pay at most $V > 0$ for a low quality brand, and θ for a high quality brand, where θ is distributed uniformly along the interval $[V, V + 1]$. Suppose that V is sufficiently high such that all consumers will buy from one of the firms.

Under full information, consumers can perfectly observe the identity of each firm. Given the prices of H and L, p_H and p_L, respectively, consumers have positive utility from buying L as long as $V > p_L$, and are willing to buy H instead of L as long as $\theta - p_H > V - p_L$, or $\theta > \theta_H \equiv V + p_H - p_L$. Thus if $V < \theta_H < V + 1$, consumers with $[V, \theta_H]$ will buy from L while consumers with $[\theta_H, V + 1]$ will buy from H. The demand functions for the two brands are:

$$q_L(p_L, p_H) = \begin{cases} 0, & p_H < p_L, \\ p_H - p_L, & p_L < p_H < 1 + p_L, \\ 1, & 1 + p_L < p_H, \end{cases} \qquad q_H(p_H, p_L) = \begin{cases} 1, & p_H < p_L, \\ 1 - p_H + p_L, & p_L < p_H < 1 + p_L, \\ 0, & 1 + p_L < p_H. \end{cases} \quad (1)$$

Under full information each firm sets p_i to maximize $\pi_i(p_i,p_j) = p_i\, q_i(p_i,p_j)$. The full information prices and profits are $p_H^* = 2/3$, $p_L^* = 1/3$, $\pi_H(p_H^*,p_L^*) = 4/9$ and $\pi_L(p_L^*,p_H^*) = 1/9$. Notice that firm H sets a higher price and earns higher profits than firm L.

3. Asymmetric information

Now consider asymmetric information. The structure of the information is as follows. Both firms are informed about each other's type and all consumers know that one firm has to be of higher quality than the other. I rule out the possibility that the two firms have identical qualities because in this model all consumers eventually buy from one of the firms. Therefore, the relevant question from the consumer's viewpoint is not whether or not to buy, but rather from which firm to buy, and this is affected by the question of which of the two firms sells the superior brand. Moreover, in Appendix B I show that the main results are robust in the case of four potential states: (H,L), (L,H), (H,H) and (L,L).

Suppose that $\alpha \in[0, 1]$ consumers are informed about the identity of the high quality firm, while $1 - \alpha$ consumers are uninformed. Uninformed consumers know that one of the firms has to be of higher quality than the other but they cannot a-priori distinguish between them, and view the two firms as being ex-ante identical and therefore having an equal prior probability of being the high quality. Suppose that α is exogenous and independent of θ. Within the framework of a static model considered in this paper (such that repeat purchase is irrelevant), I can provide two interpretations for the existence of informed consumers. First, Bagwell and Riorden (1991) interpret the existence of informed consumers as part of a process of diffusion of information. At the initial stage of the market, it is natural to expect that most consumers will not have the capabilities to identify the high quality firm. As the market evolves, information diffuses and more consumers become informed. For example, consumer magazines can provide valuable reviews on the brands which enable accurate comparison. Consequently, α can be a measure of the evolution of the market.[1] A second potential interpretation of the existence of informed consumers is that the high quality firm invests in comparative informative advertising. For example, the high quality firm can have a comparative test done on the two brands in a reliable laboratory and advertise the results. Given that such advertising is reliable, the more consumers are exposed to it, the more they can infer the identity of the high quality firm. Such informative advertising is clearly expensive and the high quality firm may not be able to advertise to the entire population. The high quality firm can also distribute free samples that enable consumers to familiarize themselves with the product before purchasing it. Doing so is naturally costly for firm H which may choose to supply only a fraction of the population with such samples. Under this explanation, it is possible to think of the model as a second stage, following a preliminary stage in

[1] Bagwell and Riorden (1991) identify three assumptions under which it is possible to interpret the results of the static game as a multi-period game in which α increases over time. First, each period has a unique set of consumers that enter and exit the market at the beginning and end of the period. Second, consumers cannot observe the prices from previous periods. Third, information diffusion is independent of the quantity of sales in previous periods.

which firm H invested in informative advertising or free samples which made the α consumers (who were exposed to the advertising or received the free samples) informed about its identity. I will elaborate on the implications of this second interpretation of α after solving for the separating equilibrium.

The two firms can use prices and uninformative advertising to signal their types to the uninformed consumers. Let A_i denotes the expenditure of firm of quality $i = H,L$ on uninformative advertising. This advertising is purely uninformative in that it does not increase the demand of the informed consumers and does not directly convey information to the uninformed consumers. Uninformed consumers observe a vector of signals, $((p_i,A_i),(p_j,A_j))$ and update their beliefs concerning the firms' qualities. Let $\beta((p_i,A_i),(p_j,A_j))\in[0,1]$ denote the posterior probability that firm i is of type H given that it sets (p_i,A_i) and the rival firm sets (p_j,A_j), while the posterior probability that firm j is of type H is $\beta((p_j,A_j),(p_i,A_i)) = 1 - \beta((p_i,A_i),(p_j,A_j))$. I follow Hertzendorf and Overgaard (2001) and Bontems and Meunier (forthcoming) in that the definition of $\beta((p_i,A_i),(p_j,A_j))$ takes into account that the two firms are ex-ante identical and therefore the identity of the firm sending each signal is irrelevant from the consumers' viewpoint. Intuitively, when uninformed consumers receive the same signal from the two firms, they have no reason to evaluate it differently, as they have no experience with the product of either firm. Consequently, beliefs are symmetric in that $\beta((p_i,A_i),(p_j,A_j)) = ½$ if $(p_i,A_i) = (p_j,A_j)$.[2]

The profit of a firm of quality $i = H,L$, given its strategy, (p_i,A_i), the strategy of firm $j \neq i$, and beliefs is:

$$\pi_i(p_i,p_j,A_i,\beta((p_i,A_i),(p_j,A_j))) = p_i\big(\alpha q_i(p_i,p_j) + (1 - \alpha)q_i^{UI}(p_i,p_j,\ \beta((p_i,A_i),(p_j,A_j))\big) - A_i, \qquad (2)$$

where $q_i(p_i,p_j)$ is the demand by the informed consumers, which is given by (1), and $q_i^{UI}(p_i,p_j,$ $\beta((p_i,A_i),(p_j,A_j)))$ is the demand by the uninformed consumers, on which I elaborate below. I restrict attention to pure-strategy equilibria defined as follows:

Definition 1: *A vector* $\{(p_H^{**},A_H^{**}),(p_L^{**},A_L^{**}),\beta((p_H^{**},A_H^{**}),(p_L^{**},A_L^{**}))\}$ *is a pure strategy perfect Bayesian equilibrium if:*

(a) $(p_H^{**},A_H^{**}) = \arg\max_{(p_H,A_H)} \pi_H(p_H,p_L^{**},A_H,\ \beta((p_H,\ A_H),(p_L^{**},A_L^{**})))$,

(b) $(p_L^{**},A_L^{**}) = \arg\max_{(p_L,A_L)} \pi_L(p_L,p_H^{**},\ A_L,\beta((p_L,\ A_L),(p_H^{**},A_H^{**})))$,

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[2] This assumption can also be interpreted as anonymity of the two firms.

(c) *If* $(p_H^{**},A_H^{**}) \neq (p_L^{**},A_L^{**})$ *then* $\beta((p_H^{**},A_H^{**}),(p_L^{**},A_L^{**})) = 1$ and $\beta((p_L^{**},A_L^{**}),(p_H^{**},A_H^{**})) = 0$,

(d) *If* $(p_H^{**},A_H^{**}) = (p_L^{**},A_L^{**}) = (p^{**}, A^{**})$ *then* $\beta((p^{**}, A^{**}),(p^{**}, A^{**})) = 1/2$.

Conditions (a) and (b) of Definition 1 require that each firm selects the strategy that maximizes its profit given the strategy of the other firm and given consumers' beliefs. Conditions (c) and (d), which defines separating and pooling equilibria, respectively, require that beliefs along the equilibrium path are correct, in that if in equilibrium different firms play different strategies then uninformed consumers can correctly infer the identity of the high quality firm from these strategies, whereas if both firms play the same strategies in equilibrium then these signals are uninformative and uninformed consumers remain with their prior probability beliefs. Note that since firms are ex-ante identical and beliefs are symmetric, the equilibrium is symmetric.

4. Separating equilibria

In a separating equilibrium, $(p_H^{**},A_H^{**}) \neq (p_L^{**},A_L^{**})$ and uninformed consumers infer from $((p_H^{**},A_H^{**}),(p_L^{**},A_L^{**}))$ the true identity of the high quality firm. To characterize the set of separating equilibria, I will first solve for the equilibrium (p_L^{**},A_L^{**}) given any putative (p_H^{**}, A_H^{**}), and then move to solve for the set of potential (p_H^{**}, A_H^{**}) that can be supported by a set of out-of-equilibrium beliefs as separating equilibria.

Starting with (p_L^{**}, A_L^{**}), notice that given a putative (p_H^{**}, A_H^{**}), if firm L plays (p_L^{**},A_L^{**}), beliefs are the worst possible from firm L's perspective: $\beta((p_L^{**},A_L^{**}),(p_H^{**},A_H^{**})) = 0$. Consequently, any separating equilibrium with $(p_L^{**},A_L^{**}) \neq (BR_L(p_H^{**}), 0)$, where $BR_L(p_H) = p_H/2$ is firm L's full information best response to p_H, fails, because by deviating to $(BR_L(p_H^{**}), 0)$ firm L maximizes its profit from the informed consumers, and the beliefs of uninformed consumers can only change in firm L's favor. This implies that in any separating equilibrium, given (p_H^{**}, A_H^{**}), firm L sets $(p_L^{**},A_L^{**}) = (BR_L(p_H^{**}),0)$ and earns $\pi_L\big(BR_L(p_H^{**}),p_H^{**},0,0\big) = BR_L(p_H^{**})q_L\big(BR_L(p_H^{**}),p_H^{**}\big)$.

Next consider the set of potential (p_H^{**}, A_H^{**}). A first incentive compatibility constraint on (p_H^{**}, A_H^{**}) is that given that firm H plays (p_H^{**}, A_H^{**}), firm L should not mimic firm H by playing (p_H^{**}, A_H^{**}). In such a case, uninformed consumers observe $((p_H^{**},A_H^{**}),(p_H^{**},A_H^{**}))$ and remain with their prior beliefs that each firm has equal probability of being the high quality firm. Since the two firms set the same price, all the uninformed consumers are indifferent between buying from each firm, and will randomly buy from one of them. The demand for firm L

by the uninformed consumers is therefore $q_L^{UI}(p_H^{**},p_H^{**}) = ½$. Informed consumers on the other hand observe identical prices for both the low and the high quality firm and thereby will not buy at all from firm L. Consequently, firm L earns $\pi_L\big(p_H^{**},p_H^{**},A_H^{**},½\big) = (1 - \alpha)p_H^{**} q_L^{UI}(p_H^{**},p_H^{**}) - A_H^{**}$ from such a deviation, which it will find unprofitable as long as $\pi_L\big(BR_L(p_H^{**}),p_H^{**}, 0,0\big) \geq \pi_L\big(p_H^{**},p_H^{**},A_H^{**},½\big)$, or:

$$A_H^{**} \geq A_L^S(p_H^{**}) \equiv \max\{ (1 - \alpha)p_H^{**} q_L^{UI}(p_H^{**},p_H^{**}) - BR_L(p_H^{**})q_L\big(BR_L(p_H^{**}),p_H^{**}\big), 0\}. \quad (3)$$

If a pair (p_H^{**},A_H^{**}) violates (3), then it is impossible to find out-of-equilibrium beliefs that support (p_H^{**},A_H^{**}) as a separating equilibrium. Intuitively, (3) requires that A_H^{**} should be high enough such that it will be unattractive for firm L to mimic firm H.

The second incentive compatibility constraint on (p_H^{**}, A_H^{**}) is that given that firm L plays $(BR_L(p_H^{**}), 0)$, firm H should not find it optimal to deviate to its best response given that by doing so beliefs are the worst case from firm H's viewpoint. Intuitively, if there is an alternative price, say, p_H', such that given that firm L charges $BR_L(p_H^{**})$, firm H prefers to set p_H' over (p_H^{**}, A_H^{**}) even if by doing so beliefs are the worst possible from firm H's view point, then it is impossible to find out-of-equilibrium beliefs that will deter firm H from making this deviation. The worst-case beliefs from H's viewpoint are $\beta((p_{H,}0),(BR_L(p_H^{**}),0)) = 0$ for any $p_H \neq \{p_H^{**},BR_L(p_H^{**})\}$, although symmetry still requires that $\beta((p_{H,} 0),(BR_L(p_H^{**}),0)) = ½$ for $p_H = BR_L(p_H^{**})$ because in this case uninformed consumers cannot distinguish between the two firms and thereby ascribe equal probabilities to each of the firms being the high quality one. Given this set of beliefs, if firm H sets $p_H = BR_L(p_H^{**})$, then the prices and the probabilities of being firm H are identical for the two firms. Consumers are therefore indifferent between the two firms and will randomize between them, such that the demand facing firm H by the uninformed consumers is $q_H^{UI}(p_H , BR_L(p_H^{**})) = ½$. For any $p_H \neq BR_L(p_H^{**})$, under the worst-case beliefs, uninformed consumers believe that firm H is of low quality and firm L is of high quality and the demand facing the real firm H from the uninformed consumers is $q_H^{UI}(p_H , BR_L(p_H^{**})) = q_L(p_H, BR_L(p_H^{**}))$, which is the demand for brand L defined in (1), given that the price of the presumed brand L is p_H and the price of the presumed brand H is $BR_L(p_H^{**})$. Total demand facing firm H is $\alpha q_H(p_H, BR_L(p_H^{**})) + (1 - \alpha)q_H^{UI}(p_H , BR_L(p_H^{**}))$, where $q_H(p_H, BR_L(p_H^{**}))$ is the demand from the informed consumers as given by (1). Hence, firm H's profit given these beliefs is

$$\pi_H^S(p_H, BR_L(p_H^{**}), 0, \beta((p_H, 0), (BR_L(p_H^{**}), 0)) = \begin{cases} p_H\left(\alpha + (1-\alpha)(BR_L(p_H^{**}) - p_H)\right), & p_H < BR_L(p_H^{**}), \\ p_H\left(\alpha + (1-\alpha)\frac{1}{2}\right), & p_H = BR_L(p_H^{**}), \\ p_H\left(\alpha(1 - p_H + BR_L(p_H^{**})\right), & p_H > BR_L(p_H^{**}). \end{cases} \quad (4)$$

The highest profit that firm H can make from such a deviation is $\pi^{S}{}_{H}{}^{max}(p_H{**})$ $\equiv \max_{p_H} \{\pi^S{}_H(p_H,BR_L(p_H{**}),0,\beta((p_H, 0),(BR_L(p_H{**}),0))\}$. Firm H will not deviate from the equilibrium strategies (p_H**, A_H**) as long as $\pi_H(p_H{**}, BR_L(p_H{**}),A_H{**}, 1) \geq \pi^S{}_H{}^{max}(p_H{**})$, or:

$$A_H{**} \leq A_H^S(p_H{**}) \equiv p_H{**}q_H(p_H{**},BR_L(p_H{**})) - \pi^S{}_H{}^{max}(p_H{**}). \quad (5)$$

I derive the term $\pi^S{}_H{}^{max}(p_H{**})$ in Appendix A. Note that $A_H^S(p_H)$ and $A_L^S(p_H)$ place upper and lower bounds on A_H**; thus A_H** should be high enough to deter firm L from mimicking H, and not too high because a high A_H** will deter firm H from setting its separating strategies.

After establishing the two necessary conditions for separating equilibria, I can now show that they are also sufficient.[3]

Lemma 1: *Necessary and sufficient conditions for separating equilibria are:*

i) $(p_L{**},A_L{**}) = (BR_L(p_H{**}), 0)$,

ii) $(p_H{**},A_H{**}) \in \Omega^S$, *where* $\Omega^S \equiv \{(p_H,A_H)|\ A_H^S(p_H) \geq A_H \geq A_L^S(p_H)\}$.

The set of separating equilibria, $(BR_L(p_H{**}),0)$ and $(p_H{**},A_H{**}) \in \Omega^S$, is non-empty if it is possible to find a p_H such that $A_H^S(p_H) > A_L^S(p_H)$. Straightforward calculations show that the main properties of Ω^S are as follows:

Proposition 1: *A separating equilibrium exists for any* $\alpha \in [0, 1]$. *Moreover:*

i) *If* $\alpha < \frac{1}{3}$, *then in any separating equilibrium* $A_H{**} > 0$,

ii) *If* $\alpha \geq \frac{1}{3}$, *then there are separating equilibria with both* $A_H{**} > 0$ *and* $A_H{**} = 0$,

iii) *If* $\alpha > \frac{2}{3}$, *then the full information equilibrium* $((p_H{*}, 0), (p_L{*},0))$ *is separating.*

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[3] Appendix B shows that in the 4-state case, the conditions in Lemma 1 are still sufficient for the separating equilibrium strategies in state (H,L). Moreover, these conditions are also necessary if $\alpha \in [1/2, 1]$.

The first part of Proposition 1 shows that in the initial stage of the industry (i.e., when most consumers are uninformed about the identity of the high quality firm), advertising is necessary for signaling quality. Intuitively, if the proportion of the informed consumers is small, then p_H alone is a blunt instrument for signaling quality because the loss of informed consumers that firm *L* will have to incur by mimicking the price of firm *H* is negligible. The second part of Proposition 1 shows that this problem vanishes for $\alpha > 1/3$, in which case there are separating equilibria without advertising. The third part of Proposition 1 shows that in the mature stage of the industry, even though some consumers are still uninformed, the loss of informed consumers that firm *L* will have to incur by mimicking firm *H*'s price is sufficiently large for firm *H* to be able to signal quality with no distortion at all.

Proposition 1 provides initial insight concerning the set of separating equilibria, but nonetheless some of these equilibria can only be supported by unreasonable out-of-equilibrium beliefs that can be ruled out. I therefore turn to refine the set of separating equilibria. Such equilibrium refinement should take into account the competitive nature of the game, in that consumers observe two signals from different firms on the same state.

To this end, suppose that a belief function sustains only one equilibrium. I make this assumption because in multi-sender signaling games, a belief function may potentially sustain more than one equilibrium. For example, consumers may expect that given that firm *L* sets the separating strategies (p_L^{**},A_L^{**}), firm *H* should set (p_H^{**},A_H^{**}) and given that firm *L* sets the pooling strategies (p^{**},A^{**}), firm *H* should also set the same pooling strategies, thus the same belief function may sustain both $((p_H^{**},A_H^{**}),(p_L^{**},A_L^{**}))$ and $((p^{**},A^{**}),(p^{**},A^{**}))$ as equilibria. However, I rule out such a possibility because it invokes the unreasonable feature that consumers expect the two firms to be able to coordinate on one of several equilibria that are supported by their beliefs, even though they set their strategies simultaneously and non-cooperatively.[4]

With a belief function that sustains only one separating equilibrium, suppose that one of the firms deviates from the equilibrium signals, while the other firm's signals are consistent with a certain separating equilibrium. What should uninformed consumers believe when they observe such a unilateral deviation? Since the other firm plays the separating equilibrium strategies, it is reasonable to expect that consumers can learn the firms' private information from the signals of the firm that did not deviate, and thereby ignore the unilateral deviation. This is the logic behind the *unprejudiced beliefs* refinement introduced by Bagwell and Ramey (1991). Indeed, in a series of laboratory experiments based on Bagwell and Ramey's paper, Müller et al. (2007) find that by

[4] I thank an anonymous referee for raising this point.

and large, a unilateral deviation from the equilibrium signals has a negligible effect on beliefs, although beliefs changed dramatically following a large unilateral deviation, in particular if the deviation is to a strategy that is not part of any other separating equilibrium. Following this logic, I adopt the restricted version of unprejudiced beliefs introduced by Hertzendorf and Overgaard (2001), according to which a unilateral deviation has no effect on beliefs as long as this deviation is to signals that belong to another separating equilibrium.[5] More precisely:

Definition 2: *Consider a separating equilibrium with* $(BR_L(p_H^{**}),0)$ *and* $(p_H^{**},A_H^{**})\in \Omega^S$. *Then, this equilibrium is resistant to equilibrium defection (REDE) if for any* $(p',A') \neq (p_H^{**},A_H^{**})$, $\beta((p',A'),(BR_L(p_H^{**}), 0)) = 1$ *if* $(p',A') \in \Omega^S$.

Intuitively, consider a separating equilibrium with $((p_H^{**},A_H^{**}), (BR_L(p_H^{**}),0))$. Suppose that firm *H* deviates by playing some (p',A'). If (p',A') is an equilibrium strategy for firm *H* in another separating equilibrium, then consumers should ignore this unilateral deviation and continue to believe that the firm that played (p',A') is the high quality firm. A separating equilibrium survives these beliefs and is therefore REDE, if firm *H* does not find it optimal to deviate from (p_H^{**},A_H^{**}) to any $(p',A') \in \Omega^S$, even though doing so still signals that it is the high quality firm. In order to prevent such deviation by firm *H*, any separating equilibrium which is REDE has to satisfy:

$$i)\quad (p_H^{**}, A_H^{**}) = \arg\max_{(p_H, A_H)} \pi_H(p_H, p_L^{**}, A_H, 1) \qquad (6)$$

$$\text{s.t.}\quad (p_H,A_H) \in \Omega^S$$

$$ii)\quad (p_L^{**},A_L^{**}) = (BR_L(p_H^{**}),0)$$

The first condition in (6) implies that if there is some $(p',A')\in\Omega^S$ such that given $(BR_L(p_H^{**}),0)$, firm *H* can earn higher profits by playing (p',A') rather than the separating strategies (p_H^{**},A_H^{**}), then firm *H* will deviate to (p',A') and the equilibrium fails. The second condition in (6) implies that if firm *L* does not play its full information best response to the equilibrium strategies of firm *H*, then firm *L* has nothing to lose from deviating and again the equilibrium fails. Taking the two conditions together reveals that the REDE refinement only sustains the less costly separating strategies. That is, given the equilibrium strategy of its rival, in the REDE equilibrium each firm chooses the separating strategy that maximizes its profit among the set of potential separating

[5] This refinement was later employed by Bontems and Meunier (forthcoming).

strategies. Let $p_H^{**}(\alpha)$, $A_H^{**}(\alpha)$ and $p_L^{**}(\alpha)$ denote the solution to (6). I can now state the main result of this paper: [6]

Proposition 2: *There is a unique separating equilibrium which is REDE for all* $\alpha \in [0,1]$. *Moreover:*

- *i)* *For* $0 \le \alpha < 1/3$, $p_H^{**}(\alpha) < p_H^*$, *and* $p_H^{**}(\alpha)$ *is increasing with* α *while* $A_H^{**}(\alpha) > 0$ *and* $A_H^{**}(\alpha)$ *is decreasing with* α;
- *ii)* *For* $1/3 \le \alpha < 3/5$, $p_H^{**}(\alpha) > p_H^*$, *and* $p_H^{**}(\alpha)$ *is increasing with* α *while* $A_H^{**}(\alpha) > 0$ *and* $A_H^{**}(\alpha)$ *is decreasing with* α;
- *iii)* *For* $3/5 \le \alpha < 2/3$, $p_H^{**}(\alpha) > p_H^*$, *and* $p_H^{**}(\alpha)$ *is decreasing with* α *and* $A_H^{**}(\alpha) = 0$;
- *iv)* *For* $2/3 \le \alpha \le 1$, $p_H^{**}(\alpha) = p_H^*$ *and* $A_H^{**}(\alpha) = 0$;
- *v)* $p_L^{**}(\alpha)$ *follows an identical trend as* $p_H^{**}(\alpha)$, *and* $p_L^{**}(\alpha) < p_H^{**}(\alpha)$ *for all* $\alpha \in [0,1]$.

The results of Proposition 2 are illustrated in Figure 1. The figure shows that $p_H^{**}(\alpha)$ behaves rather differently than in the monopoly case considered by Bagwell and Riorden (1991): $p_H^{**}(\alpha)$ starts below the full information equilibrium, and at first increases with α, then decreases with α until eventually $p_H^{**}(\alpha)$ reaches the full information price. Since $p_L^{**}(\alpha) = BR_L(p_H^{**}(\alpha))$, $p_L^{**}(\alpha)$ follows the same trend as $p_H^{**}(\alpha)$.

To see the intuition for the results, recall from Proposition 1 that when α is low, firm H needs to invest in advertising to signal its type. Substituting (3) into (6), firm H's problem for low α is to set p_H as to maximize:

$$\pi_H(p_H, p_L, A_L^S(p_H), 1) = p_H\, q_H(p_H, p_L) - [(1 - \alpha)\, p_H q_L^{UI}(p_H, p_H) - BR_L(p_H) q_L(BR_L(p_H), p_H)], \qquad (7)$$

where the term inside the square brackets is $A_L^S(p_H)$. Thus firm H will set p_H not just to maximize its full information profit but also to reduce the expenditure on advertising. The distortion in p_H depends on how p_H affects $A_L^S(p_H)$. From the square brackets in (7), p_H has two conflicting effects on firm L's incentives to mimic H (and therefore on $A_L^S(p_H)$). The first effect is represented by the first term in the square brackets, $(1 - \alpha)p_H q_L^{UI}(p_H, p_H) = (1 - \alpha)p_H/2$, which is firm L's profit when firm L mimics H by setting the same p_H and A_H and uninformed consumers do not distinguish

[6] Appendix B shows that the REDE equilibrium strategies in state (H,L) described in Proposition 2 are also REDE in the 4-state case. Moreover, these strategies are the unique REDE for $\alpha \in [1/2, 1]$.

between the two firms (and therefore remain with their prior beliefs). This term is increasing with p_H because for a high p_H, mimicking *H* by setting the same p_H enables firm *L* to achieve collusion between the two firms in their competition for the uninformed consumers at a high price. Notice that this effect emerges because of the competition between the two firms, such that firm *L* does not mimic a monopoly of type *H* but instead mimics its actual competitor. This first effect induces firm *H* to distort p_H *downwards* in order to reduce firm *L*'s profit from mimicking *H* (which in turn reduces $A_L^S(p_H)$). The second effect is represented by the second term in the square brackets in (7), $BR_L(p_H)q_L(BR_L(p_H),p_H) = p_H^2/4$, which is firm *L*'s profit from playing its best response to p_H and therefore revealing its true type. This term is also increasing with p_H, because at a higher p_H, *L* can earn more by undercutting firm *H* and setting its best response, even though by doing so it reveals its type. This second effect induces firm *H* to distort p_H *upwards* because doing so increases firm *L*'s profit from revealing its type. Notice that the first effect becomes weaker as α increases, because mimicking *H* enables firm *L* to facilitate collusion only for competition on the uninformed consumers (informed consumers will not buy from *L* if it sets p_H). Moreover, the second effect is independent of α, because whenever firm *L* plays its separating strategies, all consumers learn its true identity. Consequently, for $\alpha < 1/3$ such that most consumers are uninformed, the first effect dominates and firm *H* distorts its price downwards. As α increases, the first effect becomes weaker, and since this effect motivates firm *H* to distort p_H downwards, this incentive also becomes weaker and firm *H* increases its price. For $1/3 < \alpha < 3/5$, the second effect becomes stronger, which motivates firm *H* to distort its price upwards, though $p_H^{**}(\alpha)$ is still increasing with α.

For $3/5 < \alpha < 2/3$, firm *L*'s motivation to mimic *H* decreases because there are less uninformed consumers to mislead. In this case, firm *H* does not invest in uninformative advertising and the effect of α on $p_H^{**}(\alpha)$ is similar to the monopoly case considered in Bagwell and Riorden (1991). Namely, at a higher α, *L* has less of an incentive to mimic *H* and $p_H^{**}(\alpha)$ is higher than under full information but declining with α. Finally, for a sufficiently high α ($\alpha > 2/3$), the separating equilibrium coincides with the full information equilibrium and is therefore independent of α.

Next I analyze the effect of an increase in α on market performance. Let

$$CS^{**}(\alpha) = \int_{V}^{p_H^{**}(\alpha)-p_L^{**}(\alpha)+V} (V - p_L^{**}(\alpha))d\theta + \int_{p_H^{**}(\alpha)-p_L^{**}(\alpha)+V}^{V+1} (\theta - p_H^{**}(\alpha))d\theta, \tag{8}$$

denote total consumer surplus in the separating equilibrium given α. Firms *H* and *L* earn $\pi_H(p_H**(\alpha), p_L**(\alpha), A_H**(\alpha),1)$ and $\pi_L(p_L**(\alpha),p_H**(\alpha),0,0)$, respectively. Social welfare is defined as the sum of consumer surplus and firms' profits.

Corollary 1: *In the REDE separating equilibrium:*

i) *The profits of both firms are increasing with* α *for* $\alpha < 3/5$, *decreasing with* α *for* $3/5 < \alpha < 2/3$ *and equals the full information profits for* $2/3 < \alpha$. *Firm H always earns a higher profit than firm L.*

ii) *Consumer surplus is decreasing with* α *for* $\alpha < 3/5$, *increasing with* α *for* $3/5 < \alpha < 2/3$ *and equals the full information surplus for* $2/3 < \alpha$.

iii) *Total social welfare is increasing with* α *for* $\alpha < 2/3$ *and equals the full information welfare for* $2/3 < \alpha$.

Intuitively, the profits of both firms follow the same trend as $p_H**(\alpha)$, because as $p_H**(\alpha)$ increases, $p_L**(\alpha) = BR_L(p_H**(\alpha))$ increases and from Proposition 2, $A_H**(\alpha)$ decreases, while consumer surplus follows the opposite trend to $p_H**(\alpha)$ because it is decreasing with both prices. As for social welfare, for $\alpha < 3/5$, both prices are increasing with α, which has a negative effect on welfare, but at the same time $A_H**(\alpha)$ is decreasing with α, which has a positive effect on welfare because $A_H**(\alpha)$ is purely uninformative and is therefore a waste of welfare. Corollary 1 shows that the second effect always dominates and therefore social welfare is increasing with α for $\alpha < 3/5$.

Corollary 1 has several interesting implications. First, Bagwell and Riorden (1991) interpret α as a measure of the evolutionary process of diffusion of information within the market, and as such α increases along time. Under this interpretation, the model predicts that in the introductory phase, product prices and profits are below their long-run (full information) levels, and as time progresses, they first increase and then decrease until they reach their long-run, full information levels. This trend was not observed by Bagwell and Riorden (1991) in the monopoly case that they considered.

Second, an alternative interpretation of α is that it measures consumers' exposure to informative advertising or free samples, which can reliably convey the true identity of the high quality firm. Under this interpretation, it is possible to think of a preliminary stage in which firm

H can invest in such informative advertising to increase α. A formal model that shows how informative advertising conveys information is beyond the scope of this paper, but since Corollary 1 shows that firm H's profit is decreasing with α for $\alpha > 3/5$, it follows that firm H will not want to make more than $\alpha = 3/5$ consumers informed, and prefer instead to keep at least some consumers uninformed in order to benefit from the strategic effect that asymmetric information has in softening price competition. Moreover, since $A_H^{**}(\alpha) > 0$ for all $\alpha < 3/5$, firm H will use a combination of informative and uninformative advertising.

One last implication of Corollary 1 is that although an increase in α can increase or decrease consumer surplus and firms' profits, it always increases social welfare. This last result indicates that under the conditions of this model, consumer magazines that make information concerning the quality of new brands common knowledge (such as the *Consumer Reports* mentioned by Bagwell and Riorden (1991)), and disclosure rules that force firms to disclose valuable information regarding their products, may have conflicting effects on consumers and firms depending on α, but are beneficial to the entire market for any given value of α.

5. Pooling equilibria

In this section I solve for the pooling equilibria of the model. In a pooling equilibrium, both firms play the same strategies, $(p_H^{**},A_H^{**}) = (p_L^{**},A_L^{**}) \equiv (p^{**}, A^{**})$. Uninformed consumers learn nothing from these strategies and ascribe equal probabilities to each firm being the high quality firm: $\beta((p^{**},A^{**}),(p^{**},A^{**})) = ½$. To find the set of (p^{**},A^{**}) that can be supported by a set of beliefs as a pooling equilibrium, I need to ensure that both firms prefer to set (p^{**},A^{**}).

Starting with firm L, given that firm H sets (p^{**},A^{**}), if firm L prefers to set $(BR_L(p^{**}), 0)$ instead of (p^{**},A^{**}) even if doing so reveals that it is the low quality firm, then it is impossible to find out-of-equilibrium beliefs that prevent firm L from making this deviation. Therefore, the first restriction on (p^{**},A^{**}) is that firm L will not find such deviation profitable. If firm L sets (p^{**},A^{**}), then the demand for firm L from the uninformed consumers is $q_L^{UI}(p^{**},p^{**}) = ½$. Informed consumers on the other hand observe identical prices for both the low and the high quality firm and will therefore not buy at all from firm L. Consequently, firm L earns $\pi_L\big(p^{**},p^{**},A^{**},½\big) = (1 - \alpha)p^{**}\, q_L^{UI}(p^{**},p^{**}) - A^{**}$. If firm L deviates to $(BR_L(p^{**}),0)$ and uninformed consumers believe that it is of low quality, the demand for firm L is given by (1) and firm L earns $\pi_L\big(BR_L(p^{**}),p^{**}, 0,0\big) = BR_L(p^{**})q_L(BR_L(p^{**}),p^{**})$. Firm L will not deviate from (p^{**},A^{**}) as long as $\pi_L\big(p^{**},p^{**},A^{**},½\big) > \pi_L\big(BR_L(p^{**}),p^{**}, 0,0\big)$, or:

$$A^{**} < A_L^P(p^{**}) \equiv (1 - \alpha)p^{**}\, q_L^{UI}(p^{**},p^{**}) - BR_L(p^{**})q_L\big(BR_L(p^{**}),p^{**}\big), 0). \qquad (9)$$

Since in a pooling equilibrium both firms set the same (p^{**},A^{**}), the same argument as above should also hold for firm H. That is, (p^{**},A^{**}) should be such that given that firm L sets (p^{**},A^{**}), firm H will not prefer to deviate to its best response even if by doing so all the uninformed consumers will believe that firm H is the low quality firm. If firm H sets (p^{**},A^{**}), all the informed consumers and half of the uninformed consumers buy from firm H and firm H earns $\pi_H(p^{**},p^{**},A^{**},½) = p^{**}(\alpha + (1 - \alpha)½) – A^{**}$. If firm H deviates from (p^{**},A^{**}) to (p_H, 0) and this deviation misleads uninformed consumers into believing that firm H is of low quality and firm L is of high quality, the demand facing the real firm H from the uninformed consumers is $q_H^{UI}(p_H\ , p^{**}) = q_L(p_H, p^{**})$, which is the demand for firm L defined in (1), given that the presumed firm L sets p_H and the presumed firm H sets p^{**}. Hence, firm H's profit given these beliefs is

$$\pi_H^P(p_H, p^{**},0,0) = \begin{cases} p_H\left(\alpha + (1-\alpha)(p^{**} - p_H)\right), & p_H < p^{**}, \\ p_H\left(\alpha(1 - p_H + p^{**}\right), & p_H \geq p^{**}. \end{cases} \qquad (10)$$

Firm H will not deviate from the pooling equilibrium (p^{**},A^{**}) as long as $\pi_H(p^{**},p^{**},A^{**},½) >$ $\pi^P{}_H{}^{max}(p^{**}) \equiv \max_{p_H} \{\pi^P{}_H(p_H,p^{**},0,0)\}$, or:

$$A^{**} \leq A^P{}_H(p^{**}) \equiv p^{**}(\alpha + (1 - \alpha)½) - \pi^P{}_H{}^{max}(p^{**}). \qquad (11)$$

Clearly, it is possible to support any (p^{**},A^{**}) that satisfies (9) and (11) as pooling by the beliefs $\beta((p,A),(p^{**},A^{**})) = 0$ for any $(p,A) \neq (p^{**},A^{**})$:

Lemma 2: *The necessary and sufficient condition for pooling equilibria is* (p^{**},A^{**}) $\in \Omega^P$, *where* $\Omega^P \equiv \{(p,A)|\ A < \min\{A_L^P(p), A_H^P(p)\}$.

Lemma 2 shows that A^{**} should be sufficiently low such the no firm will find it optimal to deviate to its best response even if by doing so all the uninformed consumers believe that it is the low quality firm. Pooling equilibria exists if the set Ω^P is non-empty.

Proposition 3: *Pooling equilibria exist only for* $0 < \alpha < 0.767$. *Moreover, any pooling price can be supported in equilibrium with or without advertising in that if* $(p^{**},A^{**}) \in \Omega^P$ *then* $(p^{**},0) \in \Omega^P$ *for all* $(p^{**},A^{**}) \in \Omega^P$.

Proposition 3 has two implications. First, recall from Proposition 1 that under separating equilibria, the firm's ability to use uninformative advertising increases the set of potential prices that can be used as separating. Under pooling equilibria, this turns out not to be the case: each pooling price can be supported as such even without the need to invest in advertising, and therefore the ability to use uninformative advertising does not benefit the two firms. Second, unlike separating equilibria which are defined for all $\alpha \in [0, 1]$, pooling equilibria do not exist if the proportion of informed consumers is sufficiently high. Intuitively, if most consumers are informed, the two firms have little to lose by deviating from a putative pooling equilibrium to their best responses, and since the two firms have different best responses, it is impossible to find out-of-equilibrium beliefs that can prevent such deviations.

In fact, the set of pooling equilibria can be further refined. To this end, note first that the REDE criteria defined in the previous section have no force in refining the set of pooling equilibria, because neither firm can free-ride on the equilibrium signal sent by the competing firm, as the pooling equilibrium signal does not reveal the identity of the high quality firm.

Therefore, consider the following version of the intuitive criterion. Suppose that instead of observing the pooling strategies, consumers observe $((p', A'),(p^{**}, A^{**}))$, where $(p', A') \neq (p^{**}, A^{**})$. Furthermore, (p', A') is such that given that the competing firm sets (p^{**},A^{**}), a firm of type H will prefer to set (p',A') over (p^{**},A^{**}) if doing so convinces uninformed consumers that it is the high quality firm, while a firm of type L would have preferred to stick to (p^{**},A^{**}) (again given that the other firm sets (p^{**},A^{**})), even if deviating to (p',A') misleads consumers into believing that it is of high quality. In this case, it is clear that consumers will believe that the deviation to (p',A') was generated by a high quality firm. More precisely:

Definition 3: *A pooling equilibrium* $(p^{**},A^{**}) \in \Omega^P$ *is intuitive if* $\beta((p',A'),(p^{**},A^{**})) = 1$ *for all* (p',A') *satisfying* $\pi_H(p',p^{**},A',1) > \pi_H(p^{**},p^{**},A^{**}, ½)$ *and* $\pi_L(p^{**},p^{**},A^{**}, ½) > \pi_L(p', p^{**},A',1)$.

Definition 3 is a generalization of the intuitive criterion to the case of competing senders, in that it requires that a deviation from a putative equilibrium (p^{**},A^{**}) to (p',A') is profitable only to the high quality firm, and that it is profitable given that the competing low quality firm continues to set the putative equilibrium (p**,A**).

A pooling equilibrium survives Definition 3 only if there is no (p', A') such that firm H benefits form deviating. To see if such (p',A') exist, let $A_H^P(p;(p^{**},A^{**}))$ and $A_L^P(p;(p^{**},A^{**}))$ denote the A that solves $\pi_H(p,p^{**},A,1) = \pi_H(p^{**},p^{**},A^{**}, ½)$ and $\pi_L(p^{**},p^{**},A^{**}, ½) = \pi_L(p, p^{**},A,1)$, respectively. Since $\pi_i(p,p^{**},A,1)$ ($i = H,L$) is decreasing with A, $\pi_H(p,p^{**},A,1) \geq \pi_H(p^{**},p^{**},A^{**}, ½)$ for any $A < A_H^P(p;(p^{**},A^{**}))$, and $\pi_L(p^{**},p^{**},A^{**}, ½) < \pi_L(p, p^{**},A,1)$ for any $A > A_L^P(p;(p^{**},A^{**}))$. A pooling equilibrium fails the intuitive criterion if $A_H^P(p;(p^{**},A^{**})) > A_L^P(p;(p^{**},A^{**}))$ because in this case it is possible to find a pair (p',A') such that $A_H^P(p';(p^{**},A^{**})) > A' > A_L^P(p';(p^{**},A^{**}))$, in which case firm H finds it optimal to deviate to (p'A') because doing so reveals its type.

Proposition 4: *All pooling equilibria that are intuitive include* $p^{**} > 1$. *No pooling equilibrium is intuitive if* $\alpha > ½$.

Since $1 > p_H^* > p_L^*$, Proposition 4 indicates that compared with the full information prices, any pooling price that survives the intuitive criterion is distorted upwards. Moreover, since $1 > p_H^{**}(\alpha) > p_L^{**}(\alpha)$, any intuitive pooling price also exceeds the separating REDE equilibrium for both firms. The intuition for these results is that in a pooling equilibrium firm H loses from the fact that uninformed consumers do not distinguish between the two firms. The intuitive criteria provide firm H with the ability to gain this forgone profit by deviating to signals (p',A') that satisfy Definition 3 and reveal its type. To prevent this deviation, the equilibrium intuitive pooling price has to be high enough to compensate firm H for the forgone profit from the uninformed consumers.

Proposition 4 also reveals that all intuitive pooling equilibria fail for $\alpha > ½$, in which case there are only separating equilibria. This result indicates that the existence of pooling equilibria does not rule out the possibility of prices that are increasing with α. To see why, recall that Proposition 2 shows that $p_H^{**}(\alpha)$ is increasing with α for $0 < \alpha < 3/5$, while intuitive pooling equilibria are possible only for $\alpha < ½$. Consequently for $½ < \alpha < 3/5$, the only reasonable equilibrium is the REDE separating equilibrium with prices that are increasing with α.[7]

[7] Appendix B shows that in the 4-state case there are no pooling equilibria if $\alpha > 1/2$, even without employing any equilibrium refinement.

6. Conclusion

The main result of this paper is that competition between two privately informed firms affects the incentives of the low quality firm to mimic the high quality in different ways. Consequently, unlike the monopoly case in which prices are decreasing with the proportion of informed consumers, under competition prices and profits are increasing with the proportion of informed consumers when this proportion is small and decreasing with the proportion of informed consumers otherwise. The results of this paper predict that, if information gradually diffuses over time, in an oligopoly a firm will signal high quality with a price which is first climbing and after a certain point declining.

However, the results are based on several simplifying assumptions that may affect their robustness. First, I have assumed that the two firms have the same marginal costs. Suppose instead that the marginal costs of producing *H* and *L* are $c_H = c > c_L = 0$ (as in Fluet and Garella (2002) and Hertzendorf and Overgaard (2002)). Notice that this assumption will not affect the minimum amount of advertising that prevents firm *L* from mimicking firm *H* under a separating equilibrium, $A_L^S(p_H)$, because from (3), $A_L^S(p_H)$ is only a function of the profit of firm *L*, and is therefore independent of the differences in the marginal costs of the two firms. Moreover, recall from (7) that in the REDE equilibrium, $A_L^S(p_H)$ is always binding on the level of uninformative advertising and the distortion in p_H depends on how p_H affects $A_L^S(p_H)$. With $A_L^S(p_H)$ being the same, the two effects of setting a high p_H as described in the text below equation (7) work in the same way, in that as α increases, the collusive effect of setting a high price decreases and $p_H^{**}(\alpha)$ will increase. For intermediate values of α in which the condition $A_L^S(p_H) = 0$ binds, the effect of α on $p_H^{**}(\alpha)$ will also be the same, because again, this effect is only a function of $A_L^S(p_H)$. However, the presence of different marginal costs will clearly change the gap between $A_L^S(p_H)$ and $A_H^S(p_H)$, as the latter will be a function of *c*, which will change the set of separating equilibria. In addition, as I explain in Appendix B, some of the results in the 4-state case are also sensitive to the assumption of identical marginal costs.

Second, I assume that there are only two competing firms. In some cases, more firms may enter the market as it evolves, in which case as α increases, the number of competing firms may increase. Such a scenario may change the results of this paper if all firms still know each other's quality. Yehezkel (2006) shows that in multi-sender signaling games with common private information (such as the one considered here), under mild conditions, the full information equilibrium is always separating if the number of senders is three or more. Thus the results of this paper depend on the assumption that the number of firms is fixed and more suitable in markets in which there are barriers to entry. Note though that the results remain robust as long as additional entry occurs only for $2/3 < \alpha < 1$, in which case the market converges to the full information prices even when only two firms compete.

A third crucial assumption is that uninformed consumers view the two firms as ex-ante identical. This assumption is suitable for the case in which uninformed consumers have no previous experience with other products produced by the two firms, or each of the two firms uses new technology on which consumers have no prior knowledge. By contrast, in markets where one firm has a better reputation than the other for producing high quality goods, say, because consumers have had good experience with previous goods produced by that firm, then whenever signals are uninformative as in the case where both firms signal that they are the high quality ones, consumers may ascribe a higher probability to the firm with the higher reputation being in fact the high quality one. The incentives of a low quality firm to misrepresent its type may be affected by such asymmetric beliefs. Intuitively, if firm *i* is of low quality but has a better reputation than firm *j*, then given that firm *j* signals that it is that high quality firm, firm *i* may have a stronger incentive to misrepresent itself and also signal that it is the high quality firm because consumers are more likely to believe firm *i* than *j*. However, by so doing firm *i* may diminish its future reputation, which may create the opposite incentive not to misrepresent its type. I believe that the issue of asymmetric beliefs in the context of this model as well as other multi-sender signaling games deserves the attention of future research.

Appendix A

Following are the proofs of Lemma 1, Corollary 1 and Propositions 1 - 4.

Proof of Lemma 1: To show that the conditions in Lemma 1 are sufficient, suppose for example that $\beta((p,A), (p_H{**},A_H{**})) = 0$ for all $(p,A) \neq (p_H{**}, A_H{**})$ and $\beta((p\ ,A), (p_L{**},A_L{**})) = 0$ for all $(p,A) \neq \{(p_H{**}, A_H{**}), (p_L{**}, A_L{**})\}$. Given that firm H sets $(p_H{**},A_H{**})$, firm L will not deviate from $(BR_L(p_H{**}), 0)$ to any other $(p,A) \neq (p_H{**}, A_H{**})$ because doing so deflects firm L from the full information best response and does not change beliefs. Firm L will also not deviate to $(p_H{**}, A_H{**})$ because of condition (3). Consequently firm L will play the separating strategy. Turning to firm H, firm H will not deviate from $(p_H{**}, A_H{**})$ to any other $(p,A) \neq (p_H{**}, A_H{**})$ because by assumption $\beta((p\ ,A), (p_L{**},A_L{**})) = 0$ and because of condition (5). Consequently firm H will play the separating equilibrium strategies. □

Proof of Proposition 1: To prove the existence of separating equilibria, I will first explicitly write (3) and (5) and then show that the set Ω^S is nonempty. Starting with (3), if firm L sets $(p_H{**},A_H{**})$ then $q_L(p_H{**},p_H{**}) = 0$ and $q_L^{UI}(p_H{**},p_H{**}) = ½$, and firm L earns $\pi_L\big(p_H{**},p_H{**},A_H{**},½\big) = (1 - \alpha)p_H{**}\ q_L^{UI}(p_H{**},p_H{**}) - A_H{**} = (1 - \alpha)\ p_H{**}/2 - A_H{*}$. If firm L sets $(BR_L(p_H{**}),0)$ then it earns $\pi_L\big(BR_L(p_H{**}),p_H{**}, 0,0\big) = p_H{**}^2/4$. Thus $A_L^S(p_H)$ is given by:

$$A_L^S(p_H) = \max\{A_1(p_H), 0\}, \quad A_1(p_H) \equiv p_H(2(1 - \alpha) - p_H)/4. \tag{A-1}$$

Next consider condition (5). If firm H sets $(p_H{**},A_H{**})$, firm H earns $\pi_H(p_H{**}, BR_L(p_H{**}),A_H{**}, 1) = p_H{**}q_H(p_H{**},BR_L(p_H{**})) - A_H{**}= p_H{**}(2 - p_H{**})/2 - A_H{*}$. If firm H deviates to any other p_H then it earns (4). Let $\pi_i(p_H, BR_L(p_H{**}))$, $i = 1,..3$, denote the term in line i of (5). To maximize (5) I use the following lemmas:

Lemma 3: *In any separating equilibrium*, $p_H{**} < 2$ *and* $BR_L(p_H{**}) < 1$.

Proof: Since in any separating equilibrium $BR_L(p_H{**}) = p_H{**}/2$, $q_H(p_H{**},p_H{**}/2) = 1 - p_H{**} + p_H{**}/2 > 0$ requires that $p_H{**} < 2$ and $BR_L(p_H{**}) = p_H{**}/2 < 1$. □

Lemma 4: *The* p_H *that maximizes* (4) *is at* $p_H \geq BR_L(p_H{**})$.

Proof: Maximizing $\pi_1(p_H, BR_L(p_H^{**}))$ yields $p_H = \min\{\alpha/2(1 - \alpha) + BR_L(p_H^{**})/2, BR_L(p_H^{**})\}$. If in this solution $p_H < BR_L(p_H^{**})$, then Firm H earns:

$$
\begin{aligned}
\pi_1(p_H, BR_L(p_H^{**})) &= p_H\big(\alpha + (1 - \alpha)(BR_L(p_H^{**}) - p_H)\big) \\
&< BR_L(p_H^{**})\big(\alpha + (1 - \alpha)(BR_L(p_H^{**}) - p_H)\big) \\
&= BR_L(p_H^{**})\big(\alpha + BR_L(p_H^{**})(1 - \alpha)\big)/2 \\
&< BR_L(p_H^{**})\big(\alpha + (1 - \alpha)\big)/2 \\
&= BR_L(p_H^{**})/2 \\
&< BR_L(p_H^{**})(1 + \alpha)/2 = \pi_2(BR_L(p_H^{**}), BR_L(p_H^{**})),
\end{aligned}
$$

where the first inequality follows because $p_H < BR_L(p_H^{**})$, the subsequent equality follows by substituting $p_H = \alpha/2(1 - \alpha) + BR_L(p_H^{**})/2$, the second inequality follows because from Lemma 3, $BR_L(p_H^{**}) < 1$ and $\alpha < 1$, and the last inequality follows because $\alpha < 1$. If the solution to $\pi_1(p_H, BR_L(p_H^{**}))$ is $p_H = BR_L(p_H^{**})$, then $\pi_1(BR_L(p_H^{**}), BR_L(p_H^{**})) = BR_L(p_H^{**})\alpha = BR_L(p_H^{**})\big((1+\alpha)/2 - (1-\alpha)/2\big) < BR_L(p_H^{**})(1 + \alpha)/2 = \pi_2(BR_L(p_H^{**}), BR_L(p_H^{**}))$, where the only inequality follows because $\alpha < 1$. Consequently, the solution to (4) is at $p_H \geq BR_L(p_H^{**})$. □

From Lemma 4, I can focus on the second and third lines in (4). Maximizing $\pi_3(p_H, BR_L(p_H^{**}))$ with respect to p_H yields $p_H = (1 + BR_L(p_H^{**}))/2 > BR_L(p_H^{**})$, where the inequality holds because $BR_L(p_H^{**}) < 1$. Thus a solution to $\pi_3(p_H, BR_L(p_H^{**}))$ always exists and firm H earns $\pi_3((1 + BR_L(p_H^{**}))/2, BR_L(p_H^{**})) = \alpha(2 + p_H^{**})^2/16$. Since $\pi_3((1 + BR_L(p_H^{**}))/2, BR_L(p_H^{**}))$ can be higher or lower than $\pi_2(BR_L(p_H^{**}), BR_L(p_H^{**}))$, I can split (4) into:

$$A_2(p_H) = p_H q_H(p_H, BR_L(p_H)) - \pi_2(BR_L(p_H), BR_L(p_H)) = p_H(3 - \alpha - 2p_H)/4, \tag{A-2}$$

$$
\begin{aligned}
A_3(p_H) &= p_H q_H(p_H, BR_L(p_H)) - \pi_3((1 + BR_L(p_H))/2, BR_L(p_H)) \\
&= p_H(2 - p_H)/2 - \alpha(2 + p_H)^2/16.
\end{aligned} \tag{A-3}
$$

Condition (5) implies that $A_H^S(p_H) < \min\{A_2(p_H), A_3(p_H)\}$.

Next I turn to compare between $A_1(p_H)$, $A_2(p_H)$ and $A_3(p_H)$. Straightforward calculation shows that $A_j(p_H) > A_i(p_H)$, $i,j = 1,\ldots,3$, for $a_{ij} < p_H < b_{ij}$, where $a_{12} = 0$, $b_{12} = 1 + \alpha$, $a_{13} = 2\alpha/(4 + \alpha)$, $b_{13} = 2$, $a_{23} = (2 - 2\sqrt{1-\alpha^2})/\alpha$ and $b_{23} = (2 + 2\sqrt{1-\alpha^2})/\alpha$. Moreover, $A_i(p_H) > 0$ for $a_i < p_H < b_i$, where $a_1 = 0$, $b_1 = 2(1 - \alpha)$, $a_2 = 0$, $b_2 = (3 - \alpha)/2$, $a_3 = 8(1 - \alpha/4 - \sqrt{1-\alpha})/(8 + \alpha)$ and $b_3 = 8(1 - \alpha/4 + \sqrt{1-\alpha})/(8 + \alpha)$. Figure 2 shows Ω^S for selected values of α. For $0 < \alpha < 1/3$, $0 < a_3 < a_{13} < a_{23} < b_{12} < b_2 < b_1 < 2 < b_{23}$, as shown in Panel (a). The figure shows that all separating equilibria includes $A_H^{**} > 0$ (because $A_1(b_{12}) > 0$). For $1/3 < \alpha < 0.828$, $0 < a_3 < a_{13} < a_{23}$ and $b_1 < b_2 < b_{12} < 2 < b_{23}$, as shown in Panel (b). The figure shows that there are separating equilibria without advertising for any $b_1 < p_H < b_2$, where $b_1 < p_H^* < b_2$ for $\alpha > 2/3$, in which case $(p_H^*,0) \in \Omega^S$. For $0.828 < \alpha < 0.836$, $0 < b_1 < a_{13} < a_3 < b_{23} < b_2 < b_3 < 2$, as shown in Panel (c). Moreover, $a_3 < p_H^* < b_2$, implying that $(p_H^*, 0) \in \Omega^S$. Finally, for $0.836 < \alpha < 1$, $0 < b_1 < a_{13} < a_3 < b_3 < b_2 < 2$, as shown in Panel (d). Since $a_3 < p_H^* < b_3$, $(p_H^*,0) \in \Omega^S$. □

Proof of Proposition 2: Solving $\max_{p_H} \{p_H q_H(p_H,p_L) - A_1(p_H)\}$ and $p_L = BR_L(p_H)$ yields $p_{H1}^{**} = (1 + \alpha)/2$. Hence, for $\alpha < 1/3$, $a_{13} < p_{H1}^{**} < b_{12}$ thereby $A_2(p_H)$ and $A_3(p_H)$ are not binding and $(p_H^{**}(\alpha),A_H^{**}(\alpha)) = (p_{H1}^{**},A_1(p_{H1}^{**}))$, where $A_1(p_{H1}^{**}) = (1 + \alpha)(3 - 5\alpha)/16$ is decreasing with α. For $1/3 < \alpha < 3/5$, $a_{13} < p_{H1}^{**} < b_1$ and again $A_2(p_H)$ and $A_3(p_H)$ are not binding and $(p_H^{**}(\alpha),A_H^{**}(\alpha)) = (p_{H1}^{**},A_1(p_{H1}^{**}))$. For $3/5 < \alpha < 2/3$, $b_1 < p_{H1}^{**} < b_2$ implying that the solution to (6) yields a corner solution with $(p_H^{**}(\alpha),A_H^{**}(\alpha)) = (b_1, 0)$, where $b_1 = 2(1 - \alpha)$. For $2/3 < \alpha < 1$ the solution to $\max_{p_H} \{p_H q_H(p_H,p_L) - 0\}$ and $p_L = BR_L(p_H)$ yields $(p_H^{**}(\alpha),A_H^{**}(\alpha)) = (2/3, 0) = (p_H^*, 0)$. The comparison of $p_H^{**}(\alpha)$ with p_H^* follows directly from Figure 1. □

Proof of Corollary 1: Substituting $p_H^{**}(\alpha)$ and $BR_L(p_H^{**}(\alpha))$ into $\pi_H(p_H^{**}(\alpha), p_L^{**}(\alpha)) - A_H^{**}(\alpha)$ and $\pi_L(p_L^{**}(\alpha),p_H^{**}(\alpha))$ yields

$$\pi_H(p_H^{**}(\alpha), BR_L(p_H^{**}(\alpha))) - A_H^{**}(\alpha) = \begin{cases} \frac{3}{16}(1+\alpha)^2, & \alpha < \frac{3}{5}, \\ 2(1-\alpha)\alpha, & \frac{3}{5} \le \alpha < \frac{2}{3}, \\ \frac{4}{9}, & \frac{2}{3} \le \alpha, \end{cases} \tag{A-4}$$

$$\pi_L(BR_L(p_H^{**}(\alpha)), p_H^{**}(\alpha)) = \begin{cases} \frac{1}{16}(1+\alpha)^2, & \alpha < \frac{3}{5}, \\ (1-\alpha)^2, & \frac{3}{5} \le \alpha < \frac{2}{3}, \\ \frac{1}{9}, & \frac{2}{3} \le \alpha. \end{cases} \tag{A-5}$$

It is straightforward to verify that both profits are increasing with α for $\alpha < 3/5$, decreasing with α for $3/5 < \alpha < 2/3$, and that $\pi_H(p_H^{**}(\alpha),BR_L(p_H^{**}(\alpha))) - A_H^{**}(\alpha) > \pi_L(BR_L(p_H^{**}(\alpha)),p_H^{**}(\alpha))$. Next, consumers' utility is inversely related to prices, which are increasing with α for $\alpha < 3/5$ and decreasing for $3/5 < \alpha < 2/3$. Finally, total social welfare can be written as

$$W^{**}(\alpha) = \int_{V}^{V+p_H^{**}(\alpha)-p_L^{**}(\alpha)} V d\theta + \int_{V+p_H^{**}(\alpha)-p_L^{**}(\alpha)}^{V+1} \theta d\theta - A_H^{**}(\alpha) = \begin{cases} V + \frac{1}{32}(9+\alpha(2+9\alpha)), & \alpha < \frac{3}{5}, \\ V + (1-\frac{1}{2}\alpha)\alpha, & \frac{3}{5} \le \alpha < \frac{2}{3} \\ V + \frac{4}{9}, & \frac{2}{3} \le \alpha, \end{cases} \quad \text{(A-6)}$$

which is increasing with α for all $\alpha < 2/3$. □

Proof of Proposition 3: To prove the existence of pooling equilibria, I will first explicitly write (9) and (11) and then show that the set Ω^P is nonempty for $\alpha < 0.767$. Starting with (9), comparing (9) with (3) reveals that $A_L^P(p) = A_L^S(p)$. Next consider (11). To solve for $\max_{p_H} \{\pi^P_H(p_H,p,0,0)\}$, define $\pi_{Hi}(p_H,p,0,0)$ as the profit at line $i = 1,2$ of (10). Maximizing the first and second lines in (10) with respect to p_H yields $p_{H1} = (\alpha/(1-\alpha) + p)/2$ and $p_{H2} = (1 + p)/2$ respectively, and firm H earns $\pi_{H1}(p_{H1},p,0,0) = (p(1-\alpha) + \alpha)^2/4(1-\alpha)$ and $\pi_{H2}(p_{H2},p,0,0) = \alpha(1 + p)^2/4$. $p_{H1} < p$ iff $p > \alpha/(1-\alpha)$ and $p_{H2} > p$ iff $p < 1$, where $1 > \alpha/(1-\alpha)$ iff $\alpha < ½$. Suppose first that $\alpha < ½$. In this case for $p > 1$ ($p < \alpha/(1-\alpha)$) the unique solution to $\max_{p_H} \{\pi^P_H(p_H,p,0,0)\}$ is the first (second) terms in (10). For $1 > p > \alpha/(1-\alpha)$, both lines of (10) have internal solutions, and the solution at the first line yields higher profits if $\pi_{H1}(p_{H1},p,0,0) > \pi_{H2}(p_{H2},p,0,0)$ or $p > \sqrt{\alpha/(1-\alpha)}$, where $1 > \sqrt{\alpha/(1-\alpha)} > \alpha/(1-\alpha)$ since $\alpha < 1/2$. Consequently, for $\alpha < ½$, firm H earns $\pi_{H1}(p_{H1},p,0,0)$ if $p > \sqrt{\alpha/(1-\alpha)}$ and $\pi_{H2}(p_{H2},p,0,0)$ otherwise. $A_H^P(p)$ is defined in this case as:

$$A_H^P(p) = \begin{cases} p(\alpha + (1-\alpha)/2) - \dfrac{(p(1-\alpha)+\alpha)^2}{4(1-\alpha)}, & p > \sqrt{\alpha/(1-\alpha)}, \\ p(\alpha + (1-\alpha)/2) - \alpha(1+p)^2/4, & p \le \sqrt{\alpha/(1-\alpha)}. \end{cases} \quad \text{(A-7)}$$

Next suppose that $\alpha > ½$ such that $1 < \alpha/(1-\alpha)$. In this case for $p > \alpha/(1-\alpha)$ ($p < 1$), the unique solution to $\max_{p_H} \{\pi^P_H(p_H,p,0,0)\}$ is the first (second) term in (10). For $\alpha/(1-\alpha) > p > 1$, both lines in (10) yields corner solutions in which firm H sets $p_H = p$ and earns $\pi_H(p,p,0,0) = \alpha p$. Consequently, for $\alpha > ½$ firm H earns $\pi_{H1}(p_{H1},p,0,0)$ if $p > \alpha/(1-\alpha)$, $\pi_H(p,p,0,0)$ if $\alpha/(1-\alpha) > p > 1$ and $\pi_{H2}(p_{H2},p,0,0)$ if $1 > p$. Thus $A_H^P(p)$ is:

$$A_H^P(p) = \begin{cases} p(\alpha + (1-\alpha)/2) - \dfrac{(p(1-\alpha)+\alpha)^2}{4(1-\alpha)}, & p > \alpha/(1-\alpha), \\ p(\alpha + (1-\alpha)/2) - p\alpha, & \alpha/(1-\alpha) > p > 1, \\ p(\alpha + (1-\alpha)/2) - \alpha(1+p)^2/4, & p \le 1. \end{cases} \quad \text{(A-8)}$$

After solving for (9) and (11), I can now turn to solve for the set Ω^P. Suppose first that $\alpha < ½$. Straightforward calculation shows that $A_L^P(p) > 0$ for $0 < p < 2(1\text{-}\alpha)$, $A_H^P(p) > 0$ for $(1\text{-}\sqrt{1-\alpha^2})/\alpha < p < 1/(1\text{-}\alpha) + \sqrt{1+\alpha}/\sqrt{1-\alpha}$ and $A_H^P(p) > A_L^P(p)$ for $0 < 1 - 1/(1+\sqrt{\alpha}) < p$, where $(1\text{-}\sqrt{1-\alpha^2})/\alpha < 1 - 1/(1+\sqrt{\alpha}) < 2(1\text{-}\alpha)$. Thus the set Ω^P is non empty and involves $p > 0$, as shown in Panel (a) of Figure 3. For $\alpha = ½$, the two lines of (A-7) coincide, and the set Ω^P is shown in Panel (b) of Figure 3. For $½ < \alpha < 0.767$, the intersection between $A_H^P(p)$ and $A_L^P(p)$ is at $p = 1 - 1/(1+\sqrt{\alpha}) < 2(1\text{-}\alpha) < 1$. Consequently, the relevant part of $A_H^P(p)$ is only the third line of (A-8). Since $A_L^P(p) > 0$ for any $0 < p < 2(1\text{-}\alpha)$, where $(1\text{-}\sqrt{1-\alpha^2})/\alpha < 1 - 1/(1+\sqrt{\alpha}) < 2(1\text{-}\alpha)$, the set Ω^P is nonempty as shown in Panel (c) of Figure 3. For $\alpha > 0.767$, $2(1\text{-}\alpha) < 1 - 1/(1+\sqrt{\alpha}) < (1\text{-}\sqrt{1-\alpha^2})/\alpha$, implying that Ω^P is empty as shown in Panel (d) of Figure 3.

Finally, note that for any $\alpha < 0.767$, $A_H^{P}{}'\big((1\text{-}\sqrt{1-\alpha^2})/\alpha\big) > 0$ and $A_L^{P}{}'\big(2(1 - \alpha)\big) < 0$, implying that any $(1\text{-}\sqrt{1-\alpha^2})/\alpha < p^{**} < 2(1\text{-}\alpha)$ is a pooling price with or without $A^{**} > 0$. □

Proof of Proposition 4: I prove Proposition 4 in the following steps. First, I show that applying the criterion, firm H will never deviate to (p, A) such that $p < p^{**}$. Second, I show that firm H will deviate to (p, A) such that $p > p^{**}$ iff $p^{**} < 1$, implying that all pooling equilibria does not satisfy Definition 3 iff $p^{**} < 1$. Third, I show that $p^{**} > 1$ iff $\alpha < ½$.

Starting in stage 1, suppose that given that firm L sets (p^{**},A^{**}), firm H deviates to (p,A) such that $p < p^{**}$ and uninformed consumers believe that it is H. Given that firm L continues to set p^{**}, firm H gains all the demand by both informed and uninformed consumers and earns $\pi_H(p,p^{**},A,1) = p - A$. Since firm H earns in the putative pooling equilibrium $\pi_H(p^{**},p^{**},A^{**}, ½) = p^{**}(\alpha + (1\text{-}\alpha)/2) - A^{**}$, $A_H^P(p,(p^{**},A^{**})) = p - p^{**}(\alpha + (1\text{-}\alpha)/2) + A^{**}$. If firm L makes the same deviation given that firm H sets (p^{**},A^{**}) and uninformed consumers believe that it is H, firm L gains all the uninformed consumers, the demand for firm L by informed consumers is given by (1) and firm L earns $\pi_L(p,p^{**},A,1) = p(\alpha(p^{**} - p) + (1\text{-}\alpha)) - A$. Since firm L earns in a putative equilibrium $\pi_L(p^{**},p^{**},A^{**}, ½) = p^{**} (1\text{-}\alpha)/2 - A^{**}$, $A_L^P(p,(p^{**},A^{**})) = p(\alpha(p^{**} - p) + (1\text{-}\alpha)) - p^{**} (1\text{-}\alpha)/2 + A^{**}$. The gap $A_H^P(p,(p^{**},A^{**})) - A_L^P(p,(p^{**},A^{**})) = \alpha(1 + p)(p - p^{**})$ equals to 0 for $p = p^{**}$, and is negative for all $p < p^{**}$, implying that $A_H^P(p,(p^{**},A^{**})) - A_L^P(p,(p^{**},A^{**})) < 0$ and there is no profitable deviation for firm H.

Next consider the second step. Suppose that given that firm L sets (p^{**},A^{**}), firm H deviates to (p, A) such that $p > p^{**}$ and that uninformed consumers believe that it is H. The demand by all consumers is given by (1) and firm H earns $\pi_H(p,p^{**},A, 1) = p(1 - p + p^{**}) - A$. Consequently $A_H^P(p,(p^{**},A^{**})) = p(1 - p + p^{**}) - p^{**}(\alpha + (1-\alpha)/2) + A^{**}$. If firm L makes the same deviation given that firm H sets (p^{**}, A^{**}), then informed consumers do not buy from it, the demand by uninformed consumers is given by (1), and firm L earns $\pi_L(p, p^{**}, A, 1) = p(1 - \alpha)(1 - p + p^{**}) - A$. Consequently, $A_L^P(p,(p^{**},A^{**})) = p(1 - \alpha)(1 - p + p^{**}) - p^{**} (1-\alpha)/2 + A^{**}$ and $A_H^P(p,(p^{**},A^{**})) - A_L^P(p,(p^{**},A^{**})) = \alpha(1 - p)(p - p^{**})$. The term in the second brackets, $(p - p^{**})$ is always positive because $p > p^{**}$. If $p^{**} > 1$, then $p > p^{**} > 1$ implying that the term in the first brackets, $(1 - p)$ is negative and therefore $A_H^P(p,(p^{**},A^{**})) - A_L^P(p,(p^{**},A^{**})) < 0$ and there is no profitable deviation for firm H. If $p^{**} < 1$, then it is possible to find a p such that $1 > p > p^{**}$, in which case the term in the first brackets, $(1 - p)$ is positive and $A_H^P(p,(p^{**},A^{**})) - A_L^P(p,(p^{**},A^{**})) > 0$ and the pooling equilibrium fails the intuitive criterion.

Next consider the third stage. From Proposition 3, in any pooling equilibria $(1-\sqrt{1-\alpha^2})/\alpha < p^{**} < 2(1 - \alpha)$. But since $(1-\sqrt{1-\alpha^2})/\alpha$ is increasing with α while $2(1 - \alpha)$ is decreasing with α, and since $(1-\sqrt{1-\alpha^2})/\alpha < 1 < 2(1 - \alpha)$ for $\alpha < ½$ and $(1-\sqrt{1-\alpha^2})/\alpha < 2(1 - \alpha) < 1$ for $\alpha > ½$, there are no pooling equilibria with $p^{**} > 1$ and no pooling equilibria survives the criterion if $\alpha > ½$. □

Appendix B

In this appendix I extend the analysis to the case of four states. Suppose that each of the two firms can be of type H or L, where firms' types are not necessarily correlated. The vector of qualities, (θ_i, θ_j), where θ_i is the quality of firm i and θ_j is the quality of firm j, can be one of four possibilities, $\{(H,H),(L,L),(H,L),(L,H)\}$, with prior probabilities $\{\gamma_{HH},\gamma_{LL},\gamma_{HL},\gamma_{LH}\}$, respectively, where $\gamma_{\theta i\theta j}\in[0,1]$ and $\gamma_{HH} + \gamma_{LL} + \gamma_{HL} + \gamma_{LH} = 1$. Maintaining the assumption of symmetry, suppose that $\gamma \equiv \gamma_{HL} = \gamma_{LH}$, such that each firm has an equal prior probability of being of type H. Consumers' utility from buying brands H and L are the same as before. Notice that the discussion in the paper is a special case of this framework in which $\gamma_{HH} = \gamma_{LL} = 0$ and $\gamma = 1/2$.

Under full information, in states (H,H) and (L,L) the two firms are of the same quality and therefore the Bertrand equilibrium prices are $p_{HH}^* = p_{LL}^* = 0$. As before, in state (H,L) (state (L,H) is equivalent) the equilibrium prices are $p_{HL}^* = 2/3$ and $p_{LH}^* = 1/3$, respectively. Clearly, no firm invests in uninformative advertising.

Under asymmetric information, I follow the previous notations by defining $\beta((p_i,A_i),(p_j,A_j))\in[0,1]$ as the uninformed consumers' posterior probability that firm *i* is of type *H* given that it sets (p_i,A_i) and the rival firm sets (p_j,A_j). However, unlike the 2-state case, in the 4-state case considered here, the posterior probability that firm *j* is of type *H*, $\beta((p_j,A_j),(p_i,A_i))$, may not necessarily be equal to 1 - $\beta((p_i,A_i),(p_j,A_j))$, because the qualities of the two firms are not negatively correlated.

In a separating equilibrium there are four pairs of prices and uninformative advertising, $\{((p_{HH}^{**}, A_{HH}^{**}),(p_{HH}^{**}, A_{HH}^{**})),((p_{LL}^{**}, A_{LL}^{**}),(p_{LL}^{**}, A_{LL}^{**})),((p_{HL}^{**}, A_{HL}^{**}),(p_{LH}^{**}, A_{LH}^{**})),((p_{LH}^{**}, A_{LH}^{**}),(p_{HL}^{**}, A_{HL}^{**}))\}$, corresponding to the four states, $\{(H,H),(L,L), (H,L), (L,H)\}$, respectively.[8] Uninformed consumers' beliefs following the equilibrium strategies are therefore $\beta((p_{HH}^{**}, A_{HH}^{**}),(p_{HH}^{**}, A_{HH}^{**})) = 1$, $\beta((p_{LL}^{**}, A_{LL}^{**}),(p_{LL}^{**}, A_{LL}^{**})) = 0$, $\beta((p_{HL}^{**}, A_{HL}^{**}),(p_{LH}^{**}, A_{LH}^{**})) = 1$ and $\beta((p_{LH}^{**}, A_{LH}^{**}),(p_{HL}^{**}, A_{HL}^{**})) = 0$.

I will start the analysis by providing some general observations that do <u>not</u> rely on the REDE refinement (or any other refinement on out-of-equilibrium beliefs).

First, notice that as in the 2-state case, in every separating equilibrium, $(p_{LH}^{**},A_{LH}^{*}) = (BR_L(p_{HL}^{**}),0)$. Intuitively, if $(p_{LH}^{**},A_{LH}^{**}) \neq (BR_L(p_{HL}^{**}),0)$, then whenever a firm of type *L* in state (*L*,*H*) plays its separating strategies, beliefs are the worst case from its viewpoint, and therefore firm *L* can always benefit from deviating towards its full information best response. However, this argument may not necessarily hold for firm *L* in state (*L*,*L*). Here, whenever firm *L* plays its equilibrium strategies, consumers' beliefs are not the worst case because the competing firm is believed to be of the same quality. Thus, firm *L* may not find it optimal to deviate from $(p_{LL}^{**}, A_{LL}^{**})$ to its full information best response because doing so may increase consumers' beliefs that the competing firm is of type *H*.

A second observation is that as in the 2-state case, the separating strategies $(p_{HL}^{**}, A_{HL}^{**})$ should satisfy the constraints (3) and (5). To see why, notice that a necessary incentive compatibility constraint on $(p_{HL}^{**}, A_{HL}^{**})$ is that in state (*H*,*L*), given that firm *H* plays $(p_{HL}^{**}, A_{HL}^{**})$, the competing firm *L* should prefer to set $(BR_L(p_{HL}^{**}),0)$ over mimicking firm *H* by playing $(p_{HL}^{**}, A_{HL}^{**})$. In the latter case, uninformed consumers observe $((p_{HL}^{**},A_{HL}^{**}),(p_{HL}^{**},A_{HL}^{**}))$. Unlike the 2-state case in which $\beta((p_{HL}^{**},A_{HL}^{**}),(p_{HL}^{**},A_{HL}^{**})) = 1/2$, with four states, consumers' beliefs can be higher or lower than 1/2, because consumers

[8]In the context of prices as signals of quality with four states, Hertzendorf and Overgaard (2002) indicate that an equilibrium can be fully separating even with two prices: $p_{LL}^{**} = p_{LH}^{**}$ and $p_{HL}^{**} = p_{HH}^{**}$. Moreover, a fully separating equilibrium may potentially have three prices: $p_{LL}^{**} = p_{LH}^{**}$ and $p_{HL}^{**} \neq p_{HH}^{**}$, or $p_{HL}^{**} = p_{HH}^{**}$ and $p_{LL}^{**} \neq p_{LH}^{**}$. Following this argument, I will assume that the separating strategies may not necessarily be distinct from one another.

may infer from these strategies that, with some probability, both firms are of type H (or both are of type L). However, applying symmetry, consumers should still believe that the two firms have the same probability of being of type H, and since prices are also the same, the two firms will equally split the demand of the uninformed consumers, which is exactly as in condition (3). A second necessary incentive compatibility constraint on (p_H**, A_H**) is that in state (H,L), given that firm L plays ($BR_L(p_H$**), 0), firm H should not find it optimal to deviate to its best response given that by doing so beliefs are the worst case from firm H's viewpoint (the intuition for this argument is the same as in Section 4). Since the worst-case beliefs for firm H are the same as in the 2-state case, namely, that firm H is in fact of low quality while the competing firm L is of high quality, the resulting condition is the same as (5).

I summarize these observations in the following Lemma:

Lemma 5: *In the 4-state case, the necessary conditions for separating equilibrium are* (p_{LH}**,A_{LH}**) = ($BR_L(p_{HL}$**), 0) *and* (p_{HL}**,A_{HL}**) $\in \Omega^S$.

Notice that the conditions in Lemma 5 are identical to those in Lemma 1, which defines the set of separating equilibria in the 2-state case. The difference between the two lemmas is that in the 4-state case, these conditions are necessary but may not be sufficient. For example, with four states, an additional condition on (p_{HL}**,A_{HL}**) should ensure that in state (H,H), one of the firms will not find it optimal to deviate from its equilibrium strategies, (p_{HH}**,A_{HH}**), to (p_{HL}**,A_{HL}**) in order to induce consumers to ascribe at least some positive probability to the event that the actual state is (H,L) and not (H,H). Likewise, an additional condition on (p_{HL}**,A_{HL}**) should ensure that firm L in state (L,H) will not deviate from its equilibrium strategy ($BR_L(p_{HL}$**), 0) to (p_{HH}**,A_{HH}**) in order to induce consumers to ascribe a positive probability to the event that the state is (H,H) and not (L,H).

A similar argument holds for any strategy combination in which each firm plays a separating strategy out of the set {(p_{HH}**, A_{HH}**),(p_{LL}**, A_{LL}**),(p_{HL}**, A_{HL}**),(p_{LH}**, A_{LH}**)}, but the two separating strategies of the two firms do not correspond to the same state (as in the example above in which the two firms play ((p_{HL}**,A_{HL}**), (p_{HH}**, A_{HH}**))). To explicitly write additional restrictions on the separating strategies that prevent such deviations, I need to define consumers' beliefs regarding such out-of-equilibrium behavior. Clearly, I cannot impose symmetry by assuming that β((p_{HL}**,A_{HL}**), (p_{HH}**, A_{HH}**)) = 1/2, because the two strategies

(p_{HL}**,A_{HL}**) and (p_{HH}**, A_{HH}**) may not be identical. Moreover, I cannot apply the REDE refinement, because this refinement only relates to the case of a unilateral deviation.[9]

In order to avoid making additional assumptions concerning out-of-equilibrium beliefs, I will start by deriving a general result that does depend on any equilibrium refinement (including the REDE refinement).

Proposition 5: *If* $\alpha \in [1/2, 1]$, *then the necessary and sufficient conditions for separating equilibria in the 4-state case are:*

i) (p_{LL}**,A_{LL}**) = ($BR_L(p_H$**), 0),

ii) (p_{HL}**,A_{HL}**) $\in \Omega^S$,

iii) (p_{HH}**,A_{HH}**) = (p_{LL}**,A_{LL}**) = (0, 0).

Moreover, conditions (i) – (iii) are sufficient for separating equilibria for all $\alpha \in [0, 1]$

Proof: I will start by showing that for $\alpha \in [1/2, 1]$, the three conditions are necessary. The first two conditions are necessary from Lemma 5. Turning to the third condition, suppose that there is a separating equilibrium with p_{HH}** > 0. Then, in the separating equilibrium in state (*H*,*H*), with equal prices and equal qualities the demand for each firm is 1/2 and each firm earns π_{HH}** = p_{HH}**/2 - A_{HH}**. Now suppose that one of the firms makes a marginal downwards price deviation and that consumers' beliefs following this deviation are that the deviating firm is of type *L* while the competing firm is of type *H*. The deviating firm loses all the sales of the uninformed consumers, but gains all the sales of the informed consumers and earns p_{HH}**α - A_{HH}** > p_{HH}**/2 - A_{HH}** = π_{HH}**, where the inequality follows because $\alpha > 1/2$ and by considering a marginal downwards deviation from p_{HH}* in the left hand side of the inequality. Thus the firm will deviate even if consumers' beliefs following such a deviation are the worst case from the firm's viewpoint, and the separating equilibrium fails. The same argument holds for any p_{LL}** > 0, because as in (*H*,*H*), in (*L*,*L*) the two firms have the same qualities. Moreover, since the only remaining candidates for separating prices are p_{HH}** = p_{LL}** = 0, the only candidates for the separating advertising are A_{HH}** = A_{LL}** = 0, because otherwise the firms earn negative profits in (*H*,*H*) and (*L*,*L*).

[9]To identify beliefs concerning out-of-equilibrium strategy combinations in the 4-state case such as ((p_{HL}**,A_{HL}**), (p_{HH}**, A_{HH}**)), Hertzendorf and Overgaard (2002) introduced a "*MIN*" criteria. Accordingly, if for example the strategies of the first firm correspond to state (*H*,*L*) while the strategies of the second firm correspond to state (*H*,*H*), consumers should ascribe zero probabilities to the states (*L*,*L*) and (*L*,*H*). Fluet and Garella (2002) assume in the proof of their Proposition 5 that for some parameter values (case (*i*)), consumers' beliefs following ((p_{HL}**,A_{HL}**), (p_{HH}**, A_{HH}**)) are that the state is (*H*,*L*), while for other parameters (case (*ii*)), consumers' beliefs are that the state is (*L*,*L*).

Next I show that these conditions are sufficient for all $\alpha \in [0, 1]$. Suppose that any unilateral deviation from ((0,0),(0,0)) does not change beliefs: $\beta((0,0),(p',A')) = \beta((p',A'),(0,0)) = \beta((0,0),(0,0))$. Given these beliefs no firm will deviate in states (*H*,*H*) and (*L*,*L*) because consumers will still believe that the two firms are of the same type, and the deviating firm will earn zero profit if $p' > 0$ and $A' = 0$, or a negative profit if $A' > 0$. As for state (*H*,*L*), no firm will deviate from its equilibrium strategies in state (*H*,*L*) to (0,0) because by doing so it earns zero profits regardless of out-of-equilibrium beliefs. Thus I can apply the same argument as in the 2-state game by assuming that $\beta((p',A'),(BR_L(p_{HL}^{**}),0)) = 0$ for any $(p',A') \neq \{(p_{HL}^{**},A_{HL}^{**}),(0,0)\}$ and $\beta((p',A'),(p_{HL}^{**},A_{HL}^{**})) = 0$ for any $(p',A') \neq \{(p_{HL}^{**},A_{HL}^{**}), (0,0)\}$. From Lemma 1, these beliefs eliminate any possibility of profitable deviation. □

Proposition 5 shows that it is always possible to support the separating strategies in state (*H*,*L*) in the 2-state case as separating in the 4-state case as well. Moreover, the 4-state case does not expand the set of potential separating strategies in state (*H*,*L*) if $\alpha \in [1/2, 1]$. In this case, the separating strategies in the two additional states, (*H*,*H*) and (*L*,*L*) are always equal to the full information prices. This last result differs from Lemma 1 in Hertzendorf and Overgaard (2002) and Proposition 5 in Fluet and Garella (2002) in two aspects. First, these authors show that there are separating equilibria in which in state (*H*,*H*), the two firms set prices above the full information prices, whereas in this model, for $\alpha \in [1/2, 1]$, all separating strategies involve playing the full information prices in state (*H*,*H*).[10] Second, they show that playing the full information strategies in state (*L*,*L*) is sufficient for separating equilibrium, whereas I show that it is also necessary. The intuition for these two differences is that in this model, if the proportion of informed consumers is sufficiently high (i.e., more than 1/2), then when the two firms are of the same quality, neither can be deterred from slightly decreasing its price below the equilibrium price of its rival, even if by doing so it loses all the sales of the uninformed consumers, because it gains all the sales of the informed consumers. This argument does not apply in Hertzendorf and Overgaard (2002) and Fluet and Garella (2002) because they focused on the case in which all consumers are uninformed. Finally, notice that since the full information prices in states (*H*,*H*) and (*L*,*L*) are the same, so are the separating prices, implying that for $\alpha \in [1/2,1]$, there are no separating equilibria in which consumers can distinguish between the two states. Nevertheless, recall that with full market coverage, consumers are only interested in the relative qualities of the

[10]Fluet and Garella (2002) show in their proof of Proposition 5 that for some parameter values the separating prices in state (*H*,*H*) can be equal to the full information prices. However, in their paper this is not a nessesery condition as in this paper.

two firms, and thus their inability to discern if the two firms have the same high or low qualities is insignificant. Moreover, this result is not robust to the assumption that marginal costs are independent of quality. If firms of type H have different marginal costs than firms of type L, then the full information prices in states (H,H) and (L,L) will differ, and consumers might be able to distinguish between the two states.

Next, I apply the REDE refinement to the 4-state case.

Proposition 6: *The set* $(p_{HL}**, A_{HL}**) = (p_H**(\alpha), A_H**(\alpha))$ *and* $(p_{HH}**, A_{HH}**) = (p_{LL}**, A_{LL}**) = (0, 0)$, *where* $(p_H**(\alpha), A_H**(\alpha))$ *are defined in Proposition 2, are an REDE separating equilibrium for all* $\alpha\in[0, 1]$, *and they are the unique REDE equilibrium for* $\alpha\in[1/2, 1]$.

Proof: First, notice that this set is indeed separating. To see why, it follows from Proposition 2 that $(p_H**(\alpha), A_H**(\alpha))\in\Omega^S$ and therefore from Proposition 5 they are separating for all α. Next, I show that this set is an REDE separating equilibrium. To this end, I first show that given any separating strategies in state (H,L), $(p_{HL}**, A_{HL}**)$, the strategies $(p_{HH}**, A_{HH}**) = (p_{LL}**, A_{LL}**) = (0, 0)$ are REDE in states (H,H) and (L,L). Consider first state (H,H). Suppose that there is a second separating equilibrium in which firms play the strategies $(p_{HH}', A_{HH}') \neq (0,0)$ in state (H,H). Moreover, suppose that a unilateral deviation from (0,0) to (p_{HH}', A_{HH}') does not change beliefs: $\beta((p_{HH}', A_{HH}'),(0,0)) = \beta((0,0),(0,0))$. Nevertheless, no firm in state (H,H) will deviate from (0,0) to (p_{HH}', A_{HH}'), because it loses all the market if $p_{HH}' > 0$, spends unnecessary resources on uninformative advertising if $A_{HH}' > 0$, or both. Thus the separating strategies $(p_{HH}**, A_{HH}**) = (0, 0)$ are REDE. Moreover, the same argument holds for $(p_{LL}**, A_{LL}**) = (0, 0)$ in state (L,L). Next consider the strategies $(p_{HL}**(\alpha), A_{HL}**(\alpha))$. Given that $(p_{HH}**, A_{HH}**) = (p_{LL}**, A_{LL}**) = (0, 0)$ is REDE, the set of separating equilibria for state (H,L) is the same as in the 2-state scenario, Ω^S. Therefore, it follows from Proposition 2 that $(p_H**(\alpha), A_H**(\alpha))$ are REDE separating strategies in state (H,L).

Finally, I show that this set is the unique REDE separating equilibrium for $\alpha\in[1/2, 1]$. From Proposition 5, if $\alpha\in[1/2, 1]$ then $(p_{HH}**, A_{HH}**) = (p_{LL}**, A_{LL}**) = (0, 0)$ is the unique separating equilibrium in states (H,H) and (L,L), and therefore by definition it is also the unique REDE. With $(p_{HH}**, A_{HH}**) = (p_{LL}**, A_{LL}**) = (0, 0)$, the set of separating strategies in state (H,L) is Ω^S, and from Proposition 2, $(p_H**(\alpha), A_H**(\alpha))$ are the unique REDE separating strategies generated from Ω^S in state (H,L). □

Proposition 6 shows that the REDE equilibrium strategies in the 2-state case are also REDE in the 4-state case. The result that this equilibrium is unique for $\alpha \in [1/2, 1]$ ensures that prices are climbing and then declining even in the 4-state case, because in the unique REDE equilibrium in state (H,L), prices are climbing for $\alpha \in [1/2, 3/5]$ and declining for $\alpha \in [3/5, 2/3]$. Proposition 6 also shows that the trend of climbing and then declining prices holds only when the two firms have different qualities. In the two states in which the firms have the same quality, (H,H) and (L,L), prices are independent of α.

Finally, I turn to pooling equilibria. I focus on two-sided pooling equilibria, in which both firms pool between states H and L. One-sided pooling equilibria in which one of the firms pools between states while the other separates violate the assumption of symmetry. Following the discussion in Section 5, the pooling strategies ((p^{**},A^{**}),(p^{**},A^{**})) should satisfy conditions (9) and (11). Otherwise, firms will deviate from the pooling strategies in state (H,L). Turning to states (H,H) and (L,L), I can use the third condition in Proposition 5 to argue that if $\alpha \in [1/2, 1]$, all pooling equilibria in which $(p^{**},A^{**}) \neq (0,0)$ fail. Intuitively, under pooling, firms in states (H,H) and (L,L) should also play (p^{**},A^{**}), but Proposition 5 shows that it is impossible to construct beliefs that prevent deviation in states (H,H) and (L,L) from $(p^{**},A^{**}) \neq (0,0)$ if α is high, because in this case each firm will always prefer to slightly reduce its price in order to attract the informed consumers, even at the cost of losing the sales of the uninformed consumers. However, the proof of Proposition 3 shows that $(p^{**},A^{**}) = (0,0)$ violates conditions (11), because for $\alpha > 1/2$, $A_H^P(p) < 0$ if $p = 0$ (see Figure 3, which illustrates the set of pooling equilibria). Therefore, all pooling equilibria fail if $\alpha \in [1/2, 1]$. I summarize this conclusion as follows:

Proposition 7: *There are no pooling equilibria in the 4-state case if* $\alpha \in [1/2, 1]$.

Proposition 7 contributes to Proposition 3 by showing that introducing an additional two states reduces the set of pooling equilibria, in that in the 2-state case all pooling equilibria fail for $\alpha \in [0.767, 1]$ while in the 4-state case they do not exist for values of α lower than 0.767. Proposition 7 differs from Proposition 4 in that the latter depends on the equilibrium refinement in Definition 3, while Proposition 7 does not rely on any equilibrium refinement.

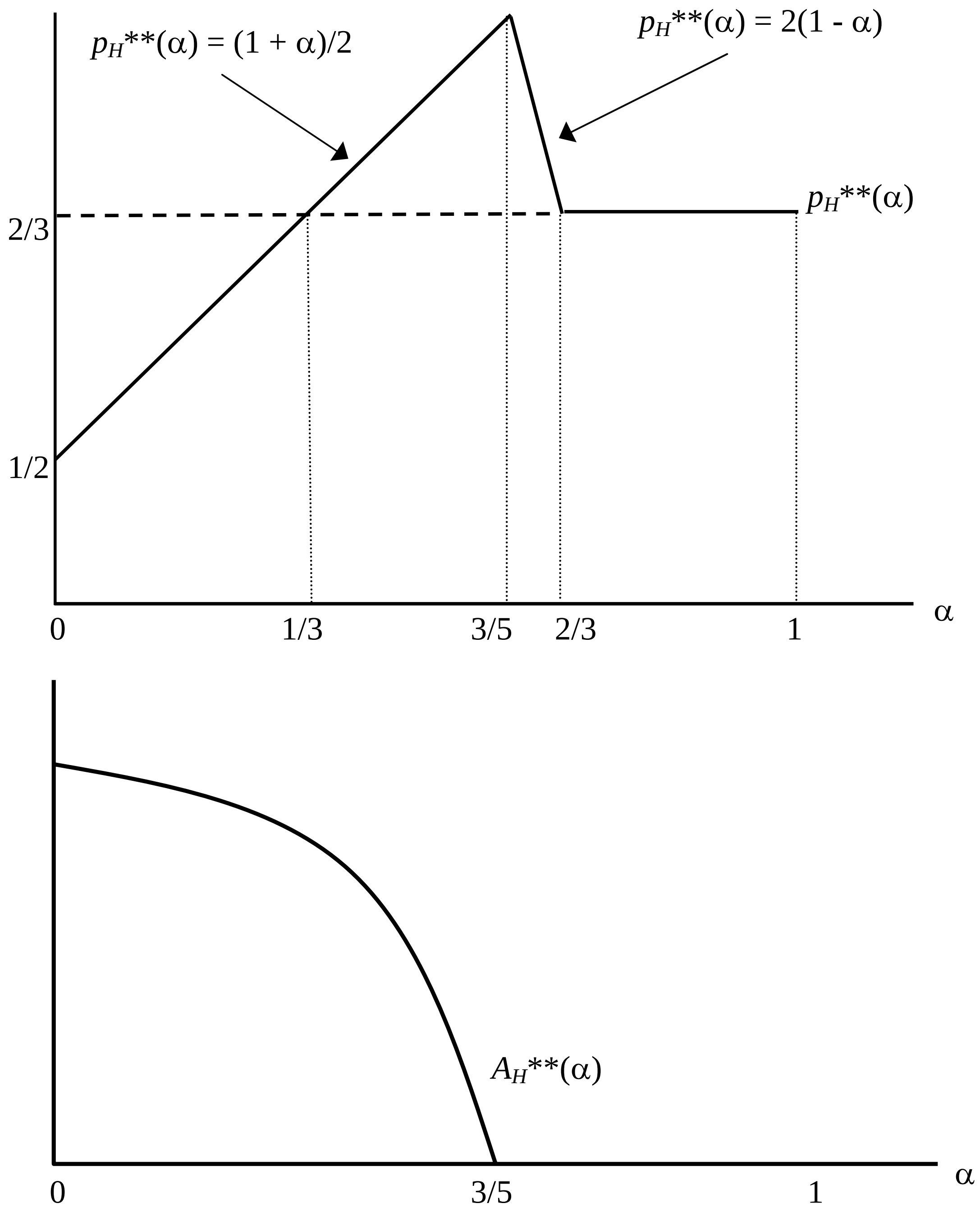

Figure 1: The separating price, p_H(α), as a function of the proportion of informed consumers, α**

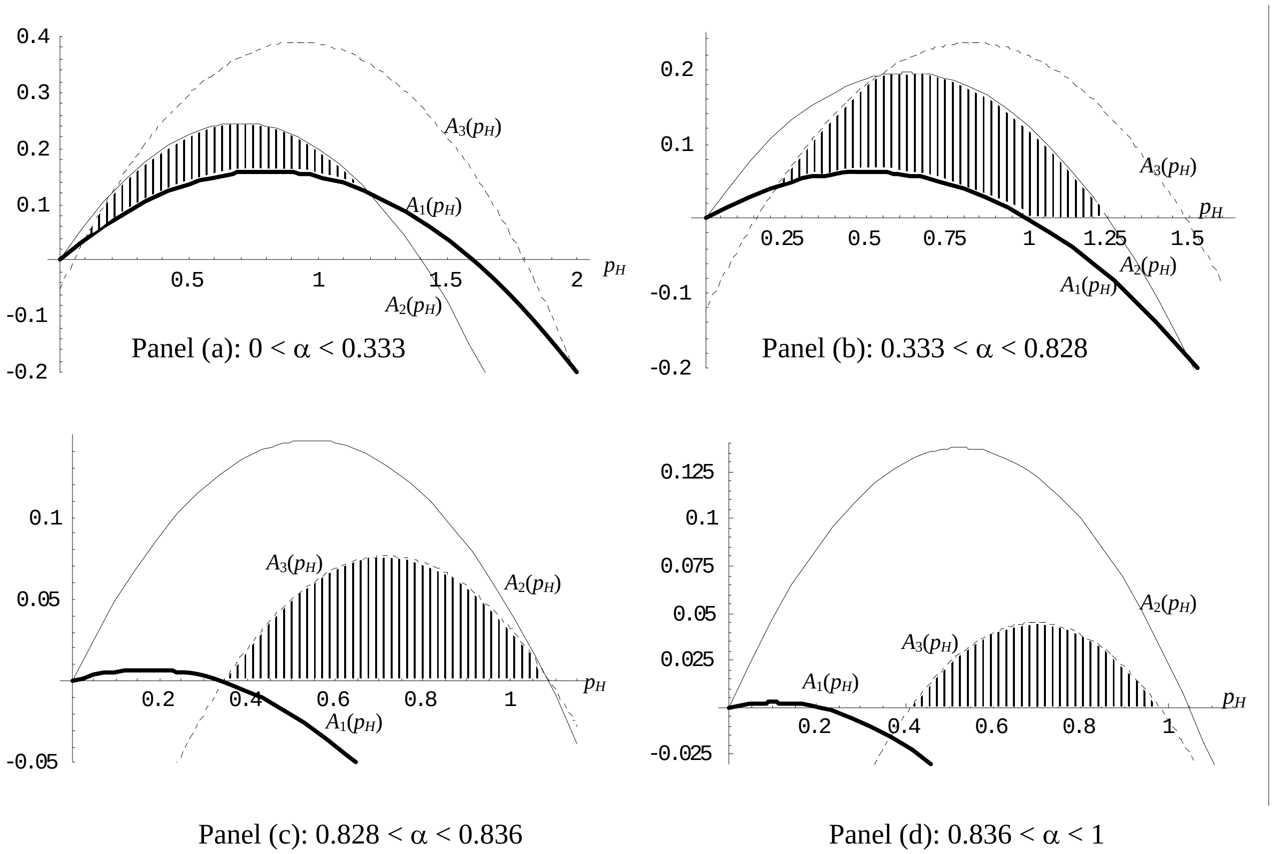

Figure 2: The set of separating equilibria

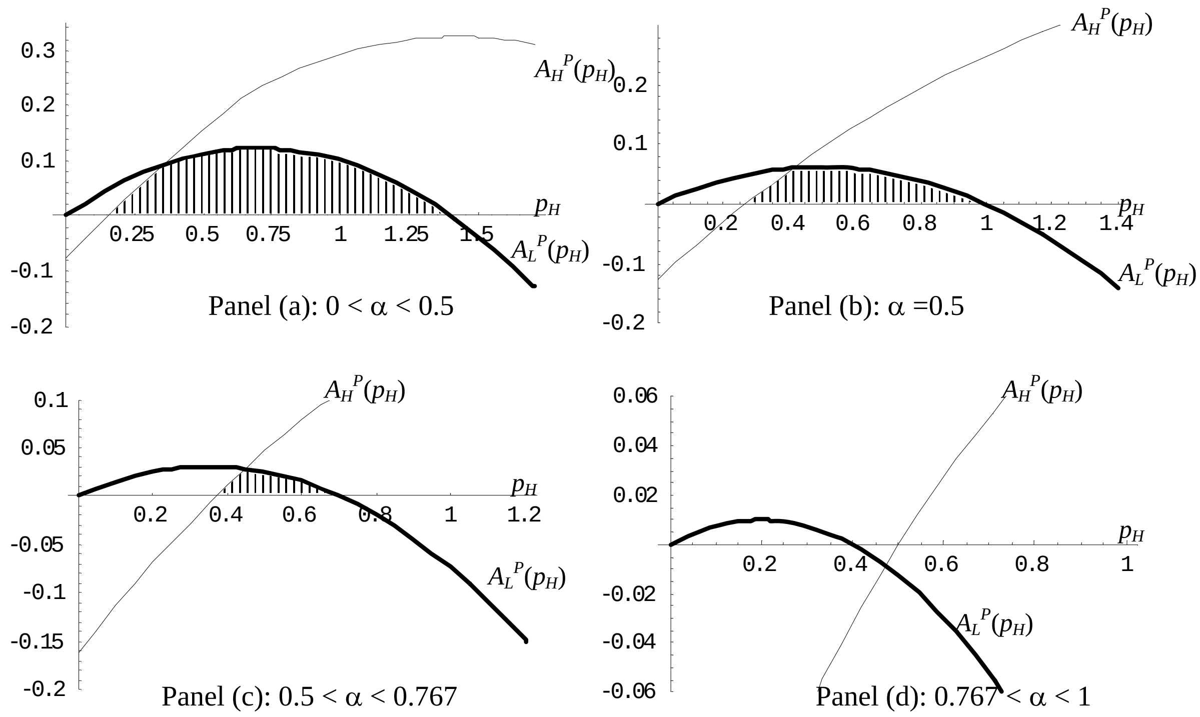

Figure 3: The set of pooling equilibria

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