Cartel Formation through Strategic Information Leakage in a Distribution Channel

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Abstract

This paper studies the ability of competing retailers to form a cartel by sharing information with their mutual manufacturer. In a market characterized by demand uncertainty, colluding retailers desire to share information about the potential market demand in order to coordinate on the optimal collusive retail price. However, since direct information-sharing between competing firms is considered to be a possible signal for collusion, according to the antitrust laws, the retailers search for a mechanism to exchange information in a manner that would not raise the suspicions of the antitrust authorities. This paper examines such a mechanism: each retailer shares his private information with the mutual manufacturer and uses the wholesale price to infer the market condition and coordinate on the cartel price. Although a cartel at the retail level limits the manufacturer's sold quantity, under certain conditions, the manufacturer is better-off accepting the retailers' private information, thereby facilitating the cartel formation. Moreover, a situation in which the retailers cannot collude by sharing information horizontally and they collude by sharing information with the manufacturer can result in a lower consumer surplus.

People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices (The Wealth of Nations, Book I, Chapter X).

1 Introduction

A considerable amount of scholarly research has examined the issue of collusion and price-fixing, when competing firms seek to establish and maintain a price that exceeds the one that would prevail in the absence of a collusive agreement. In fact, "condemnation of price-fixing agreements is not merely seen as unproblematic but as the most central, important, and defensible feature of contemporary competition law" (Kaplow 2011a: 685).

In the U.S.—as well as in most other jurisdictions—price-fixing agreements are illegal. Under the Sherman Act of 1890, parties to a collusive agreement are generally subject to harsh penalties, which may include fines and prison terms. However, while the rule against collusion and price-fixing may be "the least controversial prohibition in competition law" (Kaplow 2011b: 343), the questions regarding the precise definition of these practices and the ways they should be detected are still being debated (Kovacic 1993, Hay 2006 and Kaplow 2011b). In part, this complexity arises from the fact that direct evidence of agreements to collude is usually absent; in fact, the courts have repeatedly observed that "...seldom are the conspiratorial villains so devoid of cleverness as to broadcast their oral agreements or publicly circulate the written memos which describe their plan." [1]

Because such an agreement can rarely be demonstrated, courts have ruled that in order to prove the existence of conspiracy, an agreement under the Sherman Act need not be explicit,[2] express,[3] or formal;[4] rather it can be proved by circumstantial evidence.[5] However, in spite of the recognition of the fact that "most conspiracies are inferred from the behavior of the alleged conspirators"[6] and the overall consensus on negative welfare implications of collusion among competing firms, the detection of the behavior that constitutes collusion is exceedingly challenging, imperfect and difficult to be proven in court (Posner 1968, Kühn and Vives 1995, Posner 2001 and Kaplow 2011b).

In the absence of direct proof of conspiracy, the main challenge presented by inferring illegal collusion from the behavior of alleged conspirators lies in distinguishing it from independent decision-making by profit-maximizing firms that simply take the actions of their rivals into account. Because the actions of firms acting independently, but responding to market stimuli, may resemble the actions of conspirators (Kaplow 2011), the courts have ruled that the evidence of parallel conduct—such as parallel price movements—needs to be accompanied by evidence of some "plus factors" that enable the courts to distinguish between unilateral and

[1] Rutledge v. Electric Hose & Rubber Co., 327 F. Supp. 1267, 1274 (C.D. Cal. 1971); ES Development, 939 F.2d at 553-54 ("it is axiomatic that the typical conspiracy is 'rarely evidenced by explicit agreements,' but must almost always be proved by 'inferences that may be drawn from the behavior of the alleged conspirators' ") (quoting H.L. Moore Drug Exch. v. Eli Lilly & Co., 662 F.2d 935, 941 (2d Cir. 1981), cert. denied, 459 U.S. 880 (1982)); Todorov v. DCH Healthcare Auth., 921 F.2d 1438, 1456 (lth Cir. 1991) ("only in rare cases . . can a plaintiff establish the existence of a section 1 conspiracy by showing an explicit agreement").

[2] United States v. Gen. Motors Corp., 384 U.S. 127 (1966).

[3] United States v. Paramount Pictures, Inc., 334 U.S. 131 (1948).

[4] Am. Tobacco Co. v. United States, 328 U.S. 781 (1946).

[5] ES Development, 939 F.2d 547 (8th Cir. 1991).

[6] Seagood Trading Corp. v. Jerrrico, Inc., 924 F.2d 1555 (11th Cir. 2001).

conspiratorial conduct.[7] In *In re Flat Glass Antitrust Litigation*, the Court observed that the "[E]xistence of these plus factors tends to ensure that courts punish 'consorted action'—an actual agreement—instead of the 'unilateral, independent conduct of competitors."[8] Thus, evidence of parallel conduct without additional factors is insufficient to meet the burden of proof for finding a conspiracy.

The literature has identified a number of ways in which rival firms can coordinate their behavior in the absence of explicit agreements (Hay 1981). One such method is the exchange of private information between otherwise competing firms; "...the communication or exchange of information... might lead, through coordinated or oligopolistic interdependence, to the same results the parties sought to achieve through their proposed formal agreement" (DeSanti and Nagata 1994: 121). As a result, Kühn (2001) argues that communication between rival firms about demand information "...can be considered a restriction of competition if the (European) commission can demonstrate that no significant efficiency gains can be expected from such a scheme." Other economists, such as Athey and Bagwell (2001), Athey et al. (2004) and Gerlach (2009), explored the implications of restricting communication between competing firms, analyzing potential collusion under settings of private information and lack of communication.

Conscious of the fact that direct information-sharing may expose them to investigations by the antitrust authorities with potentially heavy penalties, competing firms seek alternative schemes to exchange information. In this paper, I focus on one such scheme, namely information-sharing between a group of retailers and their mutual manufacturer; I demonstrate that such indirect vertical information-exchange in a distribution channel can be used as a signaling device to coordinate price-fixing among competing retailers (Lee and Whang 2000). Such a collusion mechanism is hardly theoretical. For instance, the practice of using a mutual firm to facilitate collusion among otherwise competing firms received recent attention when the Department of Justice filed a civil antitrust suit against five major book publishers and Apple, Inc.[9] In this suit, the Department of Justice alleges that although each publisher has signed an independent agreement with Apple, Inc., the latter facilitated the formation of the publishers' price-fixing policy. In a different case, a bidding cartel of stamp collectors, that operated during the 1990's, employed a taxi driver that collected the internal bids from the cartel members and determined the winner of each lot. The cartel used the taxi driver in order to limit the number of meetings between the cartel members and thus reduce the risk of exposure (for a description of this cartel see Asker 2010).

In this work, I analyze a distribution channel comprised of price-setting retailers, sourcing their product from a single manufacturer, who sets the wholesale price strategically. The market demand during each period is assumed to fluctuate (similar to the seminal model of Rotemberg and Saloner 1986), and each retailer can observe a signal about the current market status. If the retailers are allowed to communicate directly, the efficient outcome, from the retailers' perspective, is achieved by aggregating their private information and establishing a monopoly price based on their cumulative knowledge.

When the risk of being exposed to antitrust investigation prevents the retailers from sharing information,

[7] For instance, see Merck-Medco Managed Care v. Rite Aid Corp., No. 98-2847, 1999 WL 691840, at *8-9 (4th Cir. Sept. 7, 1999); Blomkest Fertilizer, Inc. v. Potash Corp. of Sask., 203 F.3d 1028, (8th Cir. 2000), at 1032–1034: the Court granted defendant summary judgment because plaintiff did not meet its burden "to present evidence of consciously paralleled pricing supplemented with one or more plus factors").

[8] In re Flat Glass Antitrust Litigation, at 360 (quoting Baby Food Antitrust Litig., 166 F.3d, at 122).

[9] www.justice.gov/atr/cases/applebooks.html.

two alternative mechanisms, which result in collusion, are studied. In the first scheme, aligned with the traditional economics research about collusion with private information, each retailer sets a price based solely on his observed private information. By analyzing the ability of the retailers to collude, I demonstrate the obstacles that arise when the retailers are unable to communicate. The absence of coordination on the collusive price results in discrepancies of demand estimations; whereas some retailers infer that demand is high, others believe it to be low. This variability in the retailers' belief systems instigates a moral hazard problem; that is, despite observing a high level of market demand, a retailer may prefer to set a low price. By doing so, a retailer secures some market share for low margins, which may be preferable to incurring the risk of setting a high retail price and losing the entire market share.

As an alternative collusion mechanism, I study a scenario in which all the retailers share their private information with the manufacturer. Upon receiving the retailers' information, the manufacturer sets her wholesale price. I demonstrate how the wholesale price aggregates the information that is essential to allow the retailers to coordinate on a monopoly outcome. The wholesale price solves the problem of coordination between the retailers. Moreover, it allows the retailers to avoid the problem of price distortion and price wars that arise when the prices are based solely on the retailers' own private information.

A large body of research analyzes the incentives of firms to share private demand information in distribution channels (see, for example, Cachon and Lariviere 2001, Ozer and Wei 2006, He et al. 2008 and Guo and Iyer 2010). A few recent papers (Li 2002, Li and Zhang 2008 and Anand and Goyal 2009) have demonstrated that in an environment of competing retailers and a mutual manufacturer, the retailers may be reluctant to share their private information with the manufacturer due to the risk of this information being leaked to their rivals. In the current paper, it is the leakage phenomenon that provides the retailers—searching for tactics to communicate their private information—with an incentive to share information with the manufacturer. The retailers reveal their information to the manufacturer expecting that it will be later disclosed to their competitors through the wholesale price. To the best of my knowledge, this is the first model of collusion in which the colluding firms use their mutual manufacturer to share private information and coordinate the market price.

The model presented in this paper has important policy implications. When investigating possible cartels, antitrust authorities seek evidence of 'plus factors', such as horizontal communications and information-sharing, i.e. exchange of information between competing firms. So far, sharing information in a vertical relationship was not considered to be a practice that facilitates consort action. This paper extends the environment in which the colluding firms operate by suggesting that vertical information-sharing can serve as a tool that enables the cartel members to coordinate on a collusive outcome.

2 Literature Review

This paper contributes primarily to two streams of research: the first studies the role of communication and information sharing in cartel formation; the second examines the effect of information-leakage on the incentives to share and acquire information in a distribution channel.

Communication and Cartel Formation. Athey and Bagwell (2001) and Athey, Bagwell and Sanchirico (2004) investigated in a series of papers the scope of collusion when firms have asymmetric cost information. They showed that if the cartel strategies have to be self-enforcing and firms are sufficiently impatient, they may not be able to sustain collusion without communicating on costs. Therefore, cost communication would be a facilitating device for collusion. Gerlach (2009) demonstrated how communication improves coordination by avoiding undercutting prices by poorly informed firms. In a similar setting, Hanazono and Yang (2007) studied the possibility of competing firms to establish a cartel when they face fluctuating demand and each firm observes a signal about the market condition. Hanazano and Yang prove that absent communication, the cartel must distort its price to eliminate incentives to deviate from the cartel scheme. In the extreme case, the cartel prefers to set a rigid price and ignore the private information.

This paper differs from the economic models outlined above by extending the environment in which the colluding retailers operate. The economic models assume that if the retailers cannot share information horizontally, they must collude using sophisticated signaling games or by ignoring their private information. In this work, I assume that the retailers source their product from a mutual manufacturer and pose the question as to whether or not the retailers can collude using the manufacturer's wholesale price as a coordination device.

Harrington (2004) studied the pricing strategy of a cartel in the presence of antitrust authority. The concern of the cartel in Harrington's work is that the pricing strategy can reveal the existence of the cartel. As a result, Harrington demonstrates that the cartel can either gradually increase the pricing level or decrease its price. This work is similar to that of Harrington in the sense that the cartel members act in order to reduce the risk of exposure. However, in contrast to Harrington, the concern of the cartel members in this work is that information-sharing can be viewed as a facilitating practice and lead to exposing the cartel, and not the cartel's pricing strategy.

Another effect of information-sharing is related to the cartel's monitoring problem. Since information-exchange can enable the colluding firms to detect deviations from the collusive scheme, reducing demand uncertainty enlarges the scope of attainable collusive outcomes by increasing the efficiency of monitoring, as analyzed by Green and Porter (1984) and Abreu, Pearce and Stachetti (1986). The latter explain, "[T]he lack of transparency on prices and sales does not necessarily prevent collusion completely, but makes it both more difficult to sustain and more limited in scope". In this work I demonstrate that sharing information with the manufacturer can also reduce the uncertainty in the market and thus increase the scope of collusion.

Information Leakage and Information Sharing in Distribution Channels. The second stream of research relevant to this work is that of information leakage in vertical relationships. Information leakage describes how information reaches unintended recipients in a competitive environment. Li (2002) studies the incentives of firms to share information in a model with one supplier and n symmetric competing retailers. He shows that by observing the wholesale price, retailers who choose not to share their private information with the manufacturer can infer the information of those who did share their information. Li refers to this phenomenon as 'information leakage' and concludes that no information is shared in equilibrium— a result which is undesired by the manufacturer. Zhang (2002) studies a similar model to that of Li (2002) and adds an analysis of Bertrand competition between the retailers. Li and Zhang (2008) explore the issue of information leakage in supply chains with one manufacturer and Bertrand competing firms. They illustrate that when the retailers share their private information with the manufacturer, the retailers prefer to sign confidentiality agreements with the manufacturer. Anand and Goyal (2009) analyze the incentives of an incumbent retailer, facing possible entry to his market, to acquire new demand information when this information could be leaked to an entrant by the mutual manufacturer. They demonstrate that the manufacturer prefers to leak demand information to the entrant in order to promote competition at the retailers' level. In a recent paper, Kong et al. (2012) demonstrate that the problem of information leakage can be resolved using revenue sharing contracts between the incumbent retailer and the manufacturer.

Most of the models described above emphasize the negative effects of information leakage on the incentives of firms to share information in distribution channels. In contrast with this view, this work demonstrates that the incentives of the retailers to share their demand information with the manufacturer are driven by the retailers' hope that their private information would be leaked to their competitors; thus, enabling them to coordinate on the monopoly price. In this model, the primary goal of the retailers is to share information at the horizontal level. When this option is not available to the retailers, they choose to share their private information with the manufacturer expecting this information to later be leaked and reach the competing retailers.

This paper is also related to the literature on vertical information sharing in distribution channels. Shin and Tunca (2010) discuss the use of auctions to achieve information sharing. Ha and Tong (2008) and Ha et al. (2011) explore information sharing in two competing supply chains, and Shang et al. (2011) analyze the information sharing issue in a distribution channel comprised of one common retailer and two competing manufacturers. Guo and Iyer (2010) examine the interaction between a manufacturer's optimal information acquisition and sharing strategies in a vertical relationship. He et al. (2008) study a market in which firms have asymmetric information about the demand volatility, and they examine the potential benefits of sharing information and the contracts that facilitate such cooperation. Li (2005) considers the role of cheap talk in channel communication; he shows that communication credibility can be achieved in a vertical relationship even without any signaling costs. Gal–Or et al. (2008) study how information sharing affects wholesale pricing. These authors highlight the inference effect, which results in a manufacturer setting a low wholesale price when the retailer is uninformed. Guo (2009) highlights two effects of a retailer's information acquisition in a vertical relationship: the efficiency effect that improves retail pricing decision in a market with uncertain demand, and the strategic effect of disclosing the acquired private information to influence the manufacturer's wholesale price.

The current work also studies the incentives to share information in a vertical relationship. By sharing information, the retailers allow the manufacturer to set the wholesale price in a consistent manner with the realized demand, which in turn hurts the retailers. However, although the direct effect of information sharing results in increased cost to the retailers, this paper highlights a new motivation for information sharing with the manufacturer: by sharing information with the manufacturer, the retailers are able to signal their private information and thus establish a cartel.

3 The Model

This section describes the model. I start with a description of the events in a single period, and then embed the single period model in an infinite repeated game.

3.1 The One Period Model

Consider n retailers (I refer to each retailer as *he*) operate in a market, characterized by demand uncertainty. The retailers interact over an infinite horizon, and they source their product from a common manufacturer (*she*) for a cost of w, which is determined at the beginning of each period by the manufacturer. The demand during each period is a function of the market price $\underline{p}$ (explained below) and an i.i.d random shock:

$$Q(A_\theta, p) = (A_\theta - \underline{p})^+, \text{ where } \theta \in \{L, H\},$$

with $A_L < A_H$, and $(A_\theta - \underline{p})^+ = \max(A_\theta - \underline{p}, 0)$.[10] The probability that demand is in state $\theta = H$ is given by $\Pr(A_H) = \mu \in (0, 1)$. The profit of a monopoly at market condition A_θ setting a retail price of p and having a constant marginal cost of w is given by

$$\pi^m(A_\theta, p, w) \equiv Q(A_\theta, p)(p - w). \tag{1}$$

I denote the price a monopoly would set at market state $A_\theta \in \{A_L, A_H\}$, and with a marginal cost of w by $p^m(A_\theta, w) \equiv \arg\max_p \pi^m(A_\theta, p, w)$.

At the beginning of each selling period, each retailer has an opportunity to observe a signal $Y_i \in \{H, \phi\}$ about the market condition. The probability of observing the signal $Y_i = H$ is given by

$$\Pr(Y_i = H) = \begin{cases} \rho \text{ if } \theta = H; \\ 0 \text{ if } \theta = L, \end{cases} \tag{2}$$

and the probability of observing the signal $Y_i = \phi$ is given by

$$\Pr(Y_i = \phi) = \begin{cases} 1 - \rho \text{ if } \theta = H; \\ 1 \text{ if } \theta = L. \end{cases}$$

When the market condition is $\theta = H$, each retailer has a probability of ρ to learn the true market condition and with the complement probability a retailer observes the signal ϕ. When the market condition is $\theta = L$, a retailer can observe only the less informative signal ϕ.[11] Upon observing the signal Y_i, each retailer updates the probability that the market condition is $\theta = H$ in a Bayesian fashion.

This specific signal structure, in which upon observing the signal Y_H a retailer knows with probability 1 the market condition, was chosen in order to highlight the role of information sharing in coordinating the

[10] A linear demand curve has been widely used in models with incomplete information (e.g. Anand and Goyal 2009, Shin and Tunca 2010). In addition to its tractability, the linear demand curve has an appealing interpretation as the utility-maximizing behavior of consumers with quadratic, additively separable utility functions (Vives 1984).

[11] For convenient, I will later refer to the signal $Yi = H$ as the informative signal, and to the signal $Y_i = \phi$ as the non-informative signal since the latter provides less information about the market condition.

cartel. Under this information structure, a retailer who observes the signal Y_H, does not have an incentive to share information in order to reduce his level of uncertainty in the market; thus, his only incentive to share information is to ensure that all other cartel members set the appropriate price.

Following the information sharing stage (to be explained shortly), the manufacturer sets the wholesale price w based on her available information. At the next stage, based on their information and the wholesale price, the retailers simultaneously set the retail prices p_i, $i = 1, ..., n$. After the retailers set their prices, the market price $\underline{p}(\mathbf{p}) = \min\{\mathbf{p}\}$ is determined based on the lowest price quoted by the retailers, where $\mathbf{p} = (p_1, ..., p_n)$ denotes the vector of prices set by the retailers. The market share m_i of retailer i is given by

$$m_i(\mathbf{p}) = \begin{cases} 0 \text{ if } p_i > \underline{p}; \\ \frac{1}{k(\underline{p})} \text{ if } p_i = \underline{p}, \end{cases} \tag{3}$$

where $k(\underline{p})$ denotes the number of retailers setting the lowest price. I adopt the standard assumption that the retailers share the market equally if they set the same market price $\underline{p}$. The one period profit of retailer i is given by

$$\pi_i(A_\theta, p_i, \mathbf{p}_{-i}, w) = Q(A_\theta, \underline{p}(\mathbf{p}))(p_i - w)m_i(\mathbf{p}). \tag{4}$$

Since the retailers have identical marginal costs, the standard Bertrand game with homogenous products suggests that the stage game has a unique Nash equilibrium: all retailers charge their marginal costs regardless of the realized signals and earn zero profit in every period.

In some of the cases I analyze below, in equilibrium, each retailer can set a different price. In this case I use the superscript to denote the number of retailers setting a specific price. For example in the case $\pi_i(A_\theta, p_1^k, p_2^{n-k}, w)$, k retailers in the market set the price p_1 and $n - k$ retailers set the price p_2. I slightly abuse notation and do not use the superscript when it is clear how many retailers set the different prices in the market.

Information Structure

After observing their own private signals, the retailers have an opportunity to share information. In order to study the effect of communication on the ability of the retailers to collude I analyze a few different settings of information sharing. In the first scenario, which serves as a benchmark, I assume that communication between the retailers is allowed and I analyze the scope of collusion when the retailers can exchange information. This scenario is denoted by $S1$.

I then assume that the retailers cannot share information horizontally since such behavior results in the collusion being exposed by antitrust authorities. I examine two alternative collusion mechanisms for the retailers: in the first (denoted by $S2$), the retailers collude without any information-sharing; in the alternative option (denoted by $S3$), each retailer shares his private information vertically, i.e., solely with the manufacturer. In the last option, the retailers share their private information with the manufacturer, and no retailer is exposed to the information shared between his competitors and the manufacturer.

When n retailers share their private information, the posterior belief that the demand is high is expressed by:

	Retailers' Information	Manufacturer's Information
Setting S1	$\mathbf{Y}_n$	None
Setting S2	Y_i	None
Setting S3	Y_i and $w(\mathbf{Y})$	$\mathbf{Y}_n$

Table 1: Information available to the supply chain participants in the different settings

$$\mu(\mathbf{Y}_n) = \Pr(A_H|\mathbf{Y}_n) = \begin{cases} 1 & if \quad \text{there exists } i \text{ such that } Y_i = Y_H; \\ \frac{\mu(1-\rho)^n}{\mu(1-\rho)^n+1-\mu} & if \quad Y_i = Y_\phi \text{ for every } i, \end{cases} \tag{5}$$

where $\mathbf{Y}_n = (Y_1, Y_2, ..., Y_n)$ denotes the vector of n observed signals. I further summarize the information in the market using the information set I. I denote the state in which at least one retailer observes an informative signal by I_H, and the state in which all retailers observe the non-informative signal by I_ϕ. Finally, I use the notation $A_\mu \triangleq E[A_\theta] = \mu A_H + (1-\mu)A_L$ to denote the ex-ante value of the demand intercept, and the notation

$$\widetilde{A}_L \triangleq E[A_\theta|Y_1 = ...Y_n = \phi] = \frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{\mu(1-\rho)^n + (1-\mu)}$$

to denote the expected value of the demand intercept given that all retailers observe the non-informative signal.

Table 1 summarizes the set of information available to the different parties in the distribution-channel under the different information-exchange settings. In settings $S1$ and $S2$, the manufacturer has no information about the market demand, whereas in scenario $S3$, she receives messages from all the retailers. In scenario $S3$, although the retailers do not exchange information horizontally, they observe the wholesale price w, which may convey some information to the retailers about the market demand.

3.2 The Repeated Game

I embed the single period model in an infinitely repeated game. The market demand in each period is independent of the market demand in any other period. In each period t, the complete history of interactions between the retailers and the manufacturer, $h_t = ((w_1, \mathbf{p}_1), (w_2, \mathbf{p}_2), ..., (w_{t-1}, \mathbf{p}_{t-1}))$ is observable to both the retailers and the manufacturer. In addition, the complete history of shared information is available to the receivers of the information. Let H_t be the set of all possible period t histories. A repeated game strategy σ, consists of mapping from every possible history of actions, h_t, to the retailers' and manufacturer's one period strategy. The payoff to each retailer is the sum of the payoff during each period discounted using a common discount factor $\delta \in (0,1)$, such that the discounted profit of retailer i is expressed by:

$$V_i = E\left[\sum_{t=1}^{\infty} \delta^{t-1} \left[\pi_i(A_t, p_{it}, \mathbf{p}_{-it}, w_t)\right]\right], \tag{6}$$

and the manufacturer's discounted profit starting from period 1 is:

$$V_M = E\left[\sum_{t=1}^{\infty} \delta^{t-1}\left[(A_t - \underline{p_t})w_t)\right]\right]. \tag{7}$$

The history of the game up to date t has no direct impact on either the payoffs or the feasible strategies from date t onwards. The game beginning at date t looks the same for all t, in the sense that the feasible strategies and the prospective payoffs that they induce are always the same. History matters only because the firms remember what has happened in the past and condition their current actions on previous behavior. Infinitely repeated games typically have many equilibria (Fudenberg and Tirole 1991). As a solution concept I use the public perfect equilibria (PPE). A strategy profile is a perfect public equilibrium if the strategy played by each player depends on the public information (rather than on private information available to a specific player) and at each date t and history h_t the strategies are Nash equilibrium from that point on (i.e. no firm has a one shot profitable deviation at any point in time).

4 Collusion with Horizontal Information Sharing (Scenario $S1$)

As a benchmark, and to develop the intuition for the reminder of the results, I first analyze the collusive agreement between the retailers when they are able to exchange information. Although the manufacturer is aware of the information exchange between the retailers, she is not exposed to the shared information.

When retailers exchange information, they are all in the same information set and need to determine the set of collusive prices $\mathbf{p}^{S1} = (p_\phi^{S1}, p_H^{S1})$ for a given marginal cost w and a discount factor δ.

Let V denote the discounted expected profit of the retailers starting from the current period. When the retailers are in state I_H their profit is given by

$$V_i(I_H) = \pi_i(A_H, \mathbf{p}_H^{S1}, w) + \delta V_i. \tag{8}$$

When all retailers receive the signal Y_ϕ (i.e. they are in the information set I_ϕ) the retailers' expected profit is given by:

$$V_i(I_\phi) = \frac{(1-\mu)}{(1-\mu)+\mu(1-\rho)^n}\pi_i(A_L, \mathbf{p}_\phi^{S1}, w) + \frac{\mu(1-\rho)^n}{(1-\mu)+\mu(1-\rho)^n}\pi_i(A_H, \mathbf{p}_\phi^{S1}, w) + \delta V_i. \tag{9}$$

The discounted profit is comprised of the cartel's current period profit and the discounted stream of future profits. In the information set I_H, the retailers infer that demand is high, and they set the price p_H. In the information set I_ϕ the retailers update their belief about the state of demand and price the product to reflect the new updated belief.

For a vector of prices (p_ϕ^{S1}, p_H^{S1}) to qualify as a PPE an individual firm should have no incentive to deviate from the current period collusive price. In the case of complete information, each deviation from the collusive scheme is immediately detected and punished with the harshest possible punishment – repeatedly playing the one period Nash equilibrium and earning zero profit. Hence, the cartel's price vector (p_ϕ^{S1}, p_H^{S1}) can be sustained if the following incentive constraints are satisfied:

$$\pi_i(A_H, \mathbf{p}_H^{S1}, w) + \delta V_i \geq \pi_i(A_H, \widetilde{p}_i, \mathbf{p}_H^{S1}, w) \text{ for any } \widetilde{p}_i \neq p_H^{S1} \text{ and every retailer } i \quad \text{(IC-H)}$$

$$E_\theta[\pi_i(A_\theta, \mathbf{p}_\phi^{S1}, w)|I_\phi] + \delta V_i \geq E_\theta[\pi_i(A_\theta, \widetilde{p}_i, \mathbf{p}_\phi^{S1}, w)|I_\phi] \text{ for any } \widetilde{p}_i \neq p_\phi^{S1} \text{ and every retailer } i \quad \text{(IC-}\phi\text{)}$$

The constraint IC-H suggests that a retailer is better-off adhering to the cartel's price at the information set I_H. The Left Hand Side (LHS) denotes the expected discounted profit for a retailer setting the collusive price, given that all other retailers adhere to the cartel plan as well. The Right Hand Side (RHS) denotes the profit of deviating from the cartel scheme and setting the price $\widetilde{p}_i$. Naturally, the best deviation is to cut the cartel's price slightly and capture the entire market during the current period. Such a deviation triggers a price war and results in zero profit in all future periods. The constraint IC-ϕ is similar and ensures that a retailer does not deviate from the cartel's plan during the information set I_ϕ.

When firms are patient enough, the fear of future punishment can out-weigh the incentive to deviate from the cartel's plan and cooperation can be achieved. The following Lemma formulizes this intuition.

Lemma 1 *When information can be horizontally shared:*
(a) If $\delta \geq \overline{\delta} \triangleq \frac{n-1}{n-(1-[\mu(1-(1-\rho)^n])}$ *then*

$$(p_H, p_\phi) = \mathbf{p}^m(w);$$
$$w = \frac{A_\mu}{2}.$$

(b) No information is shared with the manufacturer.

The result demonstrates that if the discount factor is high enough, the retailers are able to set the monopoly price during each information set. Furthermore, for a discount factor higher than $\overline{\delta}$ the wholesale price does not play a role in limiting the scope of collusion; hence, the manufacturer sets the wholesale price as if she sold the product to a single monopoly.

The second part of the Lemma studies the incentives of the retailers to share information with the manufacturer when they are able to communicate horizontally. The Lemma suggests that in this case, the retailers do not have an incentive to share information with the manufacturer. Under the settings of $S1$, the retailers do not choose to voluntarily share information with the manufacturer. This result is used to demonstrate that when the retailers cannot share information horizontally (setting $S3$), their only incentive to share information with the manufacturer is to establish the cartel. In the reminder of the paper, I assume that the retailers are patient, such that a collusion according to Lemma 1 is possible.

5 Collusion without Information Sharing (Scenario $S2$)

I now turn to the analysis of the case, in which the retailers cannot share their private demand information. As discussed in the introduction, horizontal information-sharing allows antitrust authorities to infer that the retailers have colluded. As a result, the retailers search for an alternative mechanism that would allow them to collude without sharing information directly, thereby exposing them to the antitrust scrutiny. In this section, I study two mechanisms that enable the retailers to collude without sharing information directly.

In the first option, which I refer to as a *separating pricing strategy*, the retailers set different prices when they observe different signals. The second strategy discussed in this section is rigid pricing. Using this strategy, all retailers set the same price, regardless of their observed private information.

5.1 Separating Pricing Scheme

I start by analyzing the first option available to the retailers: setting a different price based on the observed signal. Denote the probability that, given high demand state, k retailers observe the non-informative signal out of n possible retailers by:

$$P_k^n \triangleq \binom{n}{k}(1-\rho)^k \rho^{n-k}. \tag{10}$$

The expected profit during the current period of a retailer observing the signal Y_H and setting the price p_H is

$$\Pi_i(p_H, Y_H) = \rho^{n-1}\pi_i(A_H, \mathbf{p}_H, w), \tag{11}$$

and the expected profit during the current period of a retailer observing the signal Y_ϕ and setting the price p_ϕ is:

$$\Pi_i(p_\phi, Y_\phi) = \frac{1-\mu}{1-\mu+\mu(1-\rho)}\pi_i(A_L, p_\phi, w) + \frac{\mu(1-\rho)}{1-\mu+\mu(1-\rho)}\sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi^{k+1}, p_H^{n-k-1}, w). \tag{12}$$

Equation (11) denotes the current period profit of a retailer observing the informative signal. When a retailer observes the informative signal, he infers that demand is high in the current period. However, setting the collusive price p_H results in profit only if all other $n-1$ retailers observe the informative signal as well. With probability $1-\rho^{n-1}$ at least one retailer observes a non-informative signal. In this case, the retailer who observes the non-informative signal sets the price p_ϕ and leaves zero profit to all the retailers who observe an informative signal. Equation (12) denotes the profit of a retailer who is observing the non-informative signal; with probability $(1-\mu)\ /\ (1-\mu+\mu(1-\rho))$ the demand during the current selling season is low; hence, all other retailers would observe the non-informative signal as well. With the probability $(\mu(1-\rho))\ /\ (1-\mu+\mu(1-\rho))$, the demand is actually high, and the retailer shares the market with all other retailers observing the non-informative signal.

For a vector of prices (p_ϕ, p_H) to be sustainable in collusion, it must be immune to two types of deviations. First, a retailer might find it beneficial to choose an *off-schedule* deviation. In this case, a retailer can choose to set a price $p \notin \{p_\phi, p_H\}$ that is different than the set of allowable prices chosen by the cartel. Such a deviation is immediately detected by all other retailers; hence, this deviation results in a price war in a manner similar to the analysis in Section 4. A second type of deviation is an *on-schedule* deviation. In this case, the retailer sets the price $p \in \{p_\phi, p_H\}$ from the set of allowable prices, but not in a manner consistent with his observed signal. The set of collusive prices (p_ϕ, p_H) is sustainable if the following set of equations is satisfied:

$$\Pi_i(p_H, Y_H) + \delta V(p_H) \geq \Pi_i(\widetilde{p}, Y_H) \text{ for every } \widetilde{p} \notin \{p_H, p_\phi\}; \quad \text{(IC-off-H)}$$

$$\Pi_i(p_\phi, Y_\phi) + \delta V(p_\phi) \geq \Pi_i(\widetilde{p}, Y_\phi) \text{ for every } \widetilde{p} \notin \{p_H, p_\phi\}; \quad \text{(IC-off-}\phi\text{)}$$

$$\Pi_i(p_H, Y_H) + \delta V(p_H) \geq \Pi_i(p_\phi, Y_H) + \delta V(p_\phi); \quad \text{(IC-on-H)}$$

$$\Pi_i(p_\phi, Y_\phi) + \delta V(p_\phi) \geq \Pi_i(p_H, Y_\phi) + \delta V(p_H). \quad \text{(IC-on-}\phi\text{)}$$

Equation (IC-off-H) denotes the off-schedule constraint for a retailer observing the informative signal. Note that by choosing a price $\widetilde{p} \notin \{p_\phi, p_H\}$, the subsequent profit of the retailer is zero since this deviation is immediately detected and punished. Equation (IC-off-ϕ) is the equivalent off-schedule constraint when the retailer observes the signal Y_ϕ. Constraints (IC-on-H) and (IC-on-ϕ) denote the on-schedule constraints when the retailer observes the informative signal and the signal Y_ϕ, respectively.

The following proposition provides some of the properties of the optimal pricing strategy of the cartel when information cannot be shared directly between the retailers.

Proposition 1 *(a) The constraints $IC-on-\phi$, $IC-off-H$ and $IC-off-\phi$ are slack in the optimal pricing strategy of the cartel.*
(b) If and only if the following condition holds:

$$\rho^{n-1}\pi_i(A_H, p_H^*, w) \geq \sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_H^{n-1-k}, p_\phi^{k+1}, w),$$

then the equilibrium cartel pricing strategy (denoted by p_H^*, p_ϕ^*) is given implicitly by the following set of equations:

$$\frac{\partial \pi_i(A_H, p_H^*, w)}{\partial p_H^*} = 0$$

$$\mu(1-\rho)\frac{\partial \pi_i(A_H, p_\phi^*, w)}{\partial p_\phi^*} + (1-\mu)\frac{\partial \pi_i(A_L, p_\phi^*, w)}{\partial p_\phi^*} = 0.$$

It was already shown in Lemma 1 that patient retailers will not adopt an off-schedule deviation. Part (a) of the Proposition supplements that when observing the signal Y_ϕ, a patient retailer will also not choose an on-schedule deviation. When a retailer, who observes the signal Y_ϕ, considers setting the price p_H, he must take into account that there is a strictly positive probability that demand is low. In this case, if a deviating retailer sets the price p_H, all other retailers infer that a deviation has occurred and the cartel collapses. As a result, a patient retailer does not choose such a deviation.

Part (b) of the Proposition characterizes the optimal pricing strategy of the colluding retailers when the condition given in Proposition 1 is satisfied. This condition suggests that a retailer observing the high signal is better-off setting the price p_H^* than mimicking a retailer observing the low signal. The condition shows that if the difference between p_H^* and p_ϕ^* is high enough, a retailer prefers to set the price p_H^* after observing the informative signal. Note that the profit of the cartel when it is able to set the prices (p_H^*, p_ϕ^*)

serves as an upper bound on the attainable cartel's profit. When the condition given in Proposition 1 is not satisfied, the cartel is not able to set the prices (p_H^*, p_ϕ^*); hence, the cartel must determine a different set of prices which results in lower profit.

When the condition given in Proposition 1 is not met, the cartel must provide the proper incentive for a retailer observing the informative signal to set the price p_H. I focus on two tools which are available to the cartel to align the incentives of a retailer observing the informative signal: price distortion and price wars (this is in addition to price rigidity, which is discussed in the next sub-section). Price distortion means that the cartel chooses a vector of prices which is different than (p_H^*, p_ϕ^*). By choosing a price $p_\phi < p_\phi^*$, the cartel reduces the incentives of a retailer observing an informative signal to mimic a retailer observing the non-informative one. The second tool available to the cartel is price wars. In this case, when some members of the cartel set the price p_ϕ during periods of high demand, it can result in initiating a price war, since there is a probability that some cartel members deviated from the cartel's plan.

In order to understand the cartel's plan when the vector of prices (p_H^*, p_ϕ^*) is not sustainable, I assume that after observing that the market demand is high and the market price was p_ϕ, the cartel collapses with probability β and the retailers set the retail price in a competitive manner for T periods. In this case, $IC - on - H$ is given by:

$$\rho^{n-1}\left[\pi_i(A_H, p_H, w) + \delta V\right] + (1-\rho^{n-1})\left[(1-\beta)\delta V + \beta\delta^{T+1}V\right] \geq \sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi^{k+1}, w) + (1-\beta)\delta V + \beta\delta^{T+1}V. \tag{14}$$

The LHS of this constraint denotes the profit of a retailer observing the informative signal and setting the price p_H. With probability ρ^{n-1} all other retailers observe the informative signal as well, and they all share the market. In this case, the cooperation continues in the next period with probability 1. However, with probability $(1 - \rho^{n-1})$, at least one retailer observes the non-informative signal; hence, a retailer observing the informative signal earns zero profit in the current period. Furthermore, the cooperation continues only with probability $(1 - \beta)$. With the complement probability, the retailers are engaged in Bertrand competition in the following T periods. The RHS of equation (14) denotes the profit of a retailer observing the informative signal, but behaving as if he had observed the non-informative signal. In this case, the deviating retailer shares the market with all other retailers observing the non-informative signal, and the cooperation continues only with probability $(1 - \beta)$. When a retailer, who observes a high signal, decides to behave as if he had observed the non-informative signal, he must take the increased probability of a price war into account and the fact that he sets a lower price compared with the price p_H.

The next proposition discusses the cartel's pricing decision when the vector of prices (p_H^*, p_ϕ^*) is not sustainable.

Proposition 2 *Assume (p_H^*, p_ϕ^*) is not sustainable in collusion. Then for any $\beta \in [0, 1]$ and $T > 0$:*
(a) $p_H = p_H^*$.
(b) $p_\phi \leq p_\phi^*$.

The proposition suggests that when the cartel cannot maintain the vector of prices (p_H^*, p_ϕ^*), it will choose to distort the prices when a retailer observes the non-informative signal, while maintaining the price

p_H^* after observing the informative signal. When the vector of prices (p_H^*, p_ϕ^*) is not sustainable, a retailer is better-off setting the price p_ϕ^* than setting the price p_H^* and facing the risk that at least one retailer observes the non-informative signal. In this case, in order to eliminate the incentives to deviate, the cartel lowers the price that a retailer sets during periods in which he observes the non-informative signal.

5.2 Rigid Pricing Scheme

As an alternative to the separating pricing scheme, the retailers can choose rigid pricing. In this case, the retailers set the same price during all selling periods, regardless of the observed signal. When the retailers set a rigid price, each deviation from this price is immediately observed and can be punished. For a high enough discount factor, the rigid price is sustainable; hence, the cartel can operate with a rigid pricing scheme over an infinite horizon.

The cartel chooses a rigid pricing scheme by solving for the problem

$$\max_p \mu(A_H - p)(p - w) + (1 - \mu)(A_L - p)(p - w).$$

The following lemma characterizes the pricing strategy and the profit of the supply chain participants in the rigid pricing scheme.

Lemma 2 *When the retailers choose a rigid pricing scheme:*

$$\begin{array}{ll} p = \frac{3}{4}A_\mu; & w = \frac{1}{2}A_\mu; \\ V_i = \frac{A_\mu^2}{16(1-\delta)}; & V_M = \frac{A_\mu^2}{8(1-\delta)}. \end{array}$$

Since there are two possible ways for the retailers to collude without sharing any information, the natural question is what the preferred way for the retailers to operate is. The next proposition shows that there are cases in which the retailers choose the rigid pricing scheme over the separating one.

Proposition 3 *There exists a value $\overline{n}$, such that for any $n \geq \overline{n}$, the profit of the cartel is higher using a rigid pricing scheme than the cartel's profit with a separating scheme.*

As the number of retailers increases, the probability of a retailer, observing a high demand signal, to capture some share of the market decreases; hence, the temptation of this retailer to deviate from the price p_H increases. In order to satisfy the incentive compatibility constraint of this retailer, the cartel is forced to distort the price p_ϕ, and be engaged in price wars. The proposition suggests that when the number of cartel members is high, in order to satisfy the incentive compatibility constraints of the cartel members, the price distortion is so significant that it is better for the cartel to ignore the private information; that is, rather than solving the coordination problem of the cartel members, the cartel prefers to use a rigid pricing scheme and ignore the private information of its members.

6 Collusion with Vertical Information Sharing (Scenario $S3$)

In this section, I study the scope of collusion when the retailers share their private information with the manufacturer. At the beginning of period t, each retailer i shares the message Y_{it} with the manufacturer. By sharing their private information with the manufacturer, the model is transformed from the setting in which the retailers have the superior information in the distribution channel into a model in which the manufacturer has the cumulative knowledge about the observed signals. Such a setting raises a few questions: what are the incentives of the retailers to share their private information with the manufacturer and how does receiving the demand information affect the wholesale price? I focus on a separating equilibrium that allows the retailers to infer the private information of the manufacturer based on the wholesale price. Although the retailers are not exposed to the private information of their competitors, they can infer this information by observing the wholesale price w. The way each retailer interprets I depends on the belief system of the retailers about the relationship between the wholesale price w and the market condition. In the subsequent analysis, I show that there exists a pair of prices w_H and w_ϕ, such that when observing w_H, the retailers infer that at least one retailer has received an informative signal, and when observing w_ϕ they infer that all the retailers have received the signal Y_ϕ. Furthermore, in equilibrium, based on this belief system, the manufacturer finds it in her best interest to set the price w_H when receiving an informative signal and the price w_ϕ when receiving only the non-informative signals.

When the retailers share their information with the manufacturer, upon observing the posted wholesale price, they establish, in equilibrium, the following belief about the set of messages observed by the manufacturer:

$$\mu(w) = \Pr(A_H|w) = \begin{cases} 1 & if \quad w = w_H; \\ \frac{\mu(1-\rho)^n}{\mu(1-\rho)^n+1-\mu} & if \quad w = w_\phi; \\ \Pr(A_H|Y_i) & if \quad w \notin \{w_\phi, w_H\}. \end{cases} \tag{15}$$

When the retailers observe the wholesale price w_H they infer that at least one retailer has observed the informative signal. When they observe the wholesale price w_ϕ they infer that all the retailers observed the non-informative signal; finally, when the manufacturer chooses to set a price which differs from w_H or w_ϕ, the retailers do not use the wholesale price to infer the market condition and each retailer uses only his private information. In a separating equilibrium, the manufacturer does not choose a wholesale price which differs from w_H or w_ϕ. Upon observing the wholesale price the retailers are able to set the collusive price as if information was shared horizontally. Based on the wholesale price, the retailers are able to solve their coordination problem; in fact, they can infer the information set and find themselves in a situation they would have been in, if they were able to meet and exchange information horizontally.

In order to provide the manufacturer with incentives to set a different wholesale price according to her observed set of signals, the following incentive constraints of the manufacturer must be satisfied.

$$\Pi_M(A_H, p_H, w_H|I_H) + \delta V_M(w_H) \geq \Pi_M(A_H, p^{S2}, \widetilde{w}|I_H) + \delta V_M^{S2} \text{ for } \widetilde{w} \notin \{w_\phi, w_H\}; \tag{M-IC-Off-H}$$

$$E\left[\Pi_M(A_i, p_\phi, w_\phi|I_\phi)\right] + \delta V_M(w_\phi) \geq E\left[\Pi_M(A_i, p^{S2}, \widetilde{w})|I_\phi\right] + \delta V_M^{S2} \text{ for } \widetilde{w} \notin \{w_\phi, w_H\} \tag{M-IC-Off-ϕ}$$

$$\Pi_M(A_H, p_H, w_H|I_H) + \delta V_M(w_H) \geq \Pi_M(A_H, p_\phi, w_\phi|I_H) + \delta V_M(w_\phi); \tag{M-IC-On-H}$$

$$E\left[\Pi_M(A_i, p_\phi, w_\phi|I_\phi)\right] + \delta V_M(w_\phi) \geq E\left[\Pi_M(A_i, p_H, w_H)|I_\phi\right] + \delta V_M(w_H); \tag{M-IC-On-ϕ}$$

In a similar manner to the set of incentive constraints the retailers have in setting $S2$, when the manufacturer possesses all the information in the market, the set of prices $\{w_\phi, w_H\}$ must be immune to two types of possible deviations. The first two constraints ensure that the manufacturer does not have an *off-schedule* incentive to deviate. In an off-schedule deviation, the manufacturer sets a wholesale price which is not included in the set $\{w_\phi, w_H\}$. The LHS denotes the profit of the manufacturer when she sets the wholesale price according to the received set of signals. The RHS denotes her profit when she sets a wholesale price which differs from w_ϕ or w_H. In this case, the retailers, who immediately observe this deviation, will choose not to share their private information with the manufacturer during all subsequent periods. Furthermore, when the retailers observe an off-schedule deviation by the manufacturer, they collude according to the analysis of setting $S2$. Hence, an off-schedule deviation is equivalent to the case in which the manufacturer refuses to act as the information aggregator and allow the retailers to coordinate on the collusive price. Since the manufacturer cannot prevent the retailers from colluding, choosing an off-schedule deviation implies that the manufacturer prefers the retailers to collude according to setting $S2$.

The second possible type of deviation a manufacturer can choose is an on-schedule deviation. In an on-schedule deviation, the manufacturer sets a price from the set of allowable prices but not according to her received signals. When the manufacturer chooses to set the price w_H when all retailers observe the non-informative signal she must take into account two effects. First, there is a strictly positive probability that this deviation would be detected by the retailers. If the realized demand is low when the manufacturer sets the price w_H, the retailers infer that the manufacturer has deviated; furthermore, they can punish her by employing the colluding mechanism that was studied in setting $S2$ and refusing to share information with the manufacturer in all subsequent selling periods. Moreover, even without the risk of exposure, this deviation is not profitable to the manufacturer. Upon learning the high wholesale price w_H, the retailers interpret it and believe the market to be good; thus, the retailers would set a high retail price, which makes the manufacturer worse-off relative to her setting the low wholesale price and inducing the retailers to set a low retail price.

I momentarily relax the off-schedule constraints and focus on characterizing an equilibrium that satisfies the on-schedule constraints of the manufacturer.[12] I will later prove that this equilibrium also satisfies the manufacturer's off-schedule constraints under mild conditions (see Proposition 5).

Before characterizing a separating equilibrium when the retailers share information vertically, an additional definition is needed. Define ψ to be:

$$\psi \triangleq \frac{A_H}{\frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{\mu(1-\rho)^n + (1-\mu)}}. \tag{16}$$

The parameter ψ can be understood as a proxy for the level of demand uncertainty in the market. A similar measure of demand uncertainty was used by Anand and Goyal (2009) and Shamir and Shin (2012).

The next proposition characterizes the separating equilibrium in the vertical information sharing scenario.

Proposition 4 *There exists an equilibrium in which*

[12] As before, I assume that since the retailers have a high discount factor they are able to set the monopoly price upon observing the wholesale price.

(a) The manufacturer sets the wholesale price

$$w_H = \frac{A_H}{2} \text{ if she receives at least one informative signal;}$$

$$w_\phi = \begin{cases} \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]} & if \quad I_\phi \text{ and } \psi \geq 3; \\ \widehat{w}_\phi & if \quad I_\phi \text{ and } \psi < 3 \end{cases}$$

where

$$\widehat{w}_\phi = \frac{2A_H - \widetilde{A}_L - \sqrt{3A_H^2 - 4A_H\widetilde{A}_L + \left(\widetilde{A}_L\right)^2}}{2}$$

(b) The retailers set the cartel price:

$$p_H = \frac{3}{4}A_H \qquad \text{if } w = w_H$$

$$p_\phi = \begin{cases} \frac{3}{4}\widetilde{A}_L & if \quad w = w_\phi \text{ and } \psi \geq 3; \\ \widehat{p}_\phi & if \quad w = w_\phi \text{ and } \psi < 3 \end{cases}$$

where

$$\widehat{p}_\phi = \frac{\widetilde{A}_L}{2} + \frac{2A_H - \widetilde{A}_L - \sqrt{3A_H^2 - 4A_H\widetilde{A}_L + \left(\widetilde{A}_L\right)^2}}{4}$$

and

$$p = p^{S2} \text{ if } w \notin \{w_\phi, w_H\},$$

with the following belief system:

$$\mu(w) = \Pr(A_H|w) = \begin{cases} 1 & if \quad w = w_H; \\ \frac{\mu(1-\rho)^n}{\mu(1-\rho)^n + 1 - \mu} & if \quad w = w_\phi; \\ \Pr(A_H|Y_i) & if \quad w \notin \{w_\phi, w_H\}. \end{cases}$$

No information is shared with the manufacturer in future periods if $w \notin \{w_\phi, w_H\}$.

In order to better understand the results of the proposition, it is beneficial to compare them to a setting of complete information. In a complete information scenario, both the manufacturer and the retailers know whether the current state is I_H or I_ϕ. In this case, the wholesale price has the operational role of maximizing the manufacturer's profit. In a complete information setting, the manufacturer would set the wholesale price to $w_H = \frac{A_H}{2}$ during the state I_H and to $w_\phi = \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]}$ during the state I_ϕ.

However, under settings of asymmetric information, when the manufacturer has superior information, the wholesale price plays a dual role: the first role is the operational role of maximizing the manufacturer's profit. The additional role, which is unique to a setting of asymmetric information, is to signal information about the demand state to the retailers. When ψ is high enough, i.e. there is a significant difference between the demand intercept during the state I_H and the expected demand intercept upon observing only non-informative signals, the manufacturer is able to set the same wholesale price as in the complete information setting. In this case, the wholesale price $w_\phi = \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]}$ conveys to the retailers, in a credible manner, that the manufacturer did not receive any informative signal. However, if ψ is not high enough (measured by $\psi < 3$) setting the complete information price during the state I_ϕ cannot convey to the retailers that indeed the manufacturer did not receive any informative signal. In this case, even if the manufacturer observed an informative signal, she has an incentive to set the price $w_\phi = \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]}$ in order to induce the retailers to lower their retail prices and thus increase her profit. Anticipating such behavior, the retailers would ignore the informative role of the wholesale price. As a result, in order to achieve a separating equilibrium, the manufacturer must design a price schedule that would convey the true state of demand to the retailers based on the wholesale price. The manufacturer achieves this goal by lowering her price during periods in which she does not receive any informative signal.

Gal-Or et al. (2008) also observe this distortion in the wholesale price under settings of asymmetric information. They define it as the *inference effect*. In their work, they show that the inference effect results in a lower wholesale price compared with the case of complete information. Proposition 4 is aligned with the findings of Gal-Or et al. (2008), showing that under settings of asymmetric information the manufacturer may need to decrease the wholesale price in order to be perceived as credible.

The Folk theorem suggests that repeated games have many equilibria; thus, the equilibrium characterized in Proposition 4 is not unique. This specific equilibrium was chosen for a few reasons. First, given the manufacturer's pricing strategy, this equilibrium provides the retailers with the highest profit, allowing me to compare the outcome of this equilibrium with the cases studied in Settings $S1$ and $S2$. Second, if $\psi \geq 3$, this equilibrium is also the optimal separating equilibrium from the manufacturer's perspective. Finally, in this equilibrium, the manufacturer's strategy is not sensitive to her discount factor. Even if the manufacturer is less patient than the retailers, the results of this equilibrium still hold. In the extreme case, even if the manufacturer has a discount factor of 0 (meaning that she does not place any weight on future profits), the pricing strategy of the manufacturer, that is characterized by Proposition 4, is still an equilibrium.

7 Comparisons between Scenarios $S2$ and $S3$

Following the analysis of sections 5 and 6, both of which allow the retailers to collude without direct information sharing, the natural question is what is the preferred mechanism from the retailers' perspective? In this section, I compare the option of the retailers to collude without communication (Setting $S2$) and the scenario in which the retailers share information with the manufacturer and infer the market condition using the wholesale price. I start by examining some of the properties of each setting.

Information Efficiency - In setting $S2$, with probability $\mu \sum_{k=1}^{n-1} P_k^n$, the cartel suffers from information inefficiency; although at least one member in the cartel observes an informative signal, there is also at least one retailer who observes the non-informative signal. In this case, the market price is set to p_ϕ^{S2} rather than

p_H^{S2}. In contrast, in setting $S3$, if at least one retailer observes an informative signal, all the other retailers can infer that demand is high through the wholesale price. Clearly, if the cartel adopts the rigid pricing scheme there is also information inefficiency, since the cartel ignores the private information of its members. The difference in the information efficiency can also lead to implications regarding information acquisition: assuming information acquisition is costly, when the cartel members cannot use their information efficiently, they may prefer not to acquire such information.

Cartel Stability - The second difference between scenario $S2$ and scenario $S3$ is the stability of the cartel. While, in scenario $S3$ the cartel can operate over infinite horizon, in scenario $S2$ (when the retailers choose a separating pricing scheme) the cartel eventually collapses, since there exists a positive probability of price wars.

Market Knowledge - The third interesting difference between the two settings is the identity of the firm in the distribution channel that has the superior information about the market demand. In scenario $S2$—when the retailers do not use price rigidity—each retailer has private information; in order to satisfy the retailer's incentive constraints, the cartel distorts (under certain conditions) the price in state I_ϕ. However, in scenario $S3$, the manufacturer has the superior information about the market demand and in order to be able to credibly signal her observed information, she may need to distort her price during the state I_ϕ.

I now return to the issue of whether scenario $S3$ is feasible. The retailers would prefer to share their private information with the manufacturer if their discounted profit in scenario $S3$ is higher than their discounted profit in scenario $S2$. Analogously, the manufacturer is willing to act as the information aggregator for the retailers if her discounted profit in scenario $S3$ is higher than the one she can obtain in scenario $S2$. Let V_R^{S2} and V_M^{S2} be the discounted profit of the typical retailer and the manufacturer, respectively, in setting $S2$. Similarly, their discounted profit in setting $S3$ is given by V_R^{S3} and V_M^{S3}. The participants in the distribution channel prefer colluding by using vertical information sharing if the following conditions hold:

$$\begin{aligned} V_R^{S3} &\geq V_R^{S2}; \\ V_M^{S3} &\geq V_M^{S2}. \end{aligned} \tag{17}$$

If both the manufacturer and the retailers prefer scenario $S3$, one can assume that it would be the natural choice for the distribution channel when the retailers try to collude without horizontal information-sharing. The next proposition demonstrates that there are cases in which the constraints given in equation (17) are satisfied.

Proposition 5 *Assume that the conditions given in Proposition 3 hold and $\psi \geq 3$. Then*

$$\begin{aligned} V_R^{S3} &\geq V_R^{S2}; \\ V_M^{S3} &\geq V_M^{S2}. \end{aligned}$$

The proposition, which is an important result in this paper, suggests that there are cases in which the manufacturer agrees to receive the information sent by the retailers, and the retailers are better-off sharing their private information with the manufacturer. Proposition 3 demonstrates that there are cases, in which the cartel chooses in scenario $S2$ the rigid pricing scheme over the separating pricing. In this case, the cartel sets the same price during all selling periods, and the cartel can operate over an infinite horizon. When this is the case, the manufacturer knows that by refusing to share information, she can expect to earn a profit of $\frac{A_{\mu}^2}{8}$ during all periods. By choosing not to share information, the manufacturer cannot break the cartel, since under the rigid pricing scheme the cartel is stable. The proposition shows that electing to share information with the manufacturer can be an equilibrium; furthermore, this equilibrium, under the conditions given in Proposition 5, Pareto dominates any other equilibrium studied in this paper. The retailers are better-off if they share information with the manufacturer than if they use either the rigid pricing scheme or the separating equilibrium studied in scenario $S2$, and the manufacturer is better-off if she accepts this information than if she earns her expected profit under scenario $S2$.

From the retailers' perspective, choosing to share information enables them to benefit from the information about demand. Choosing a rigid pricing scheme in scenario $S2$ solves the retailers' coordination problem. However, this problem is solved by ignoring the demand information available to the retailers. When the retailers share information with the manufacturer, they are able to enjoy the value of a better forecast of their demand. This proposition shows that, indeed, as Lee and Whang (2000) speculate, firms can use vertical information sharing to facilitate collusion.

Proposition 5 shows that sharing information with the manufacturer can be the preferred equilibrium when the retailers set the rigid pricing equilibrium in scenario $S2$. However, it is also interesting to compare between the performance of the separating equilibrium discussed in scenario $S2$ and the performance of the equilibrium characterized in scenario $S3$. The following proposition demonstrates that even if the retailers are able to set the most collusive price level in the separating equilibrium of scenario $S2$, there are cases in which they prefer to collude using the manufacturer.

Proposition 6 *Assume the pricing scheme for the separating strategy given in Proposition 1 is feasible. There exists $\widetilde{n}$, such that for any $n \geq \widetilde{n}$:*

$$V_R^{S3} \geq V_R^{S2};$$
$$V_M^{S3} \geq V_M^{S2}.$$

Proposition 6 suggests that there are cases in which the retailers prefer to share information with the manufacturer even when the most collusive pricing scheme is feasible in scenario $S2$. The intuition behind this result is that even when the retailers are able to set the most collusive price level and there are no price wars on the equilibrium path, there is still efficiency loss due to the fact that some retailers may observe the non-informative signal while others observe the informative one. In this case, under scenario $S2$, the retailers, observing the non-informative signal, share the market and the cartel's profit is reduced as the price these retailers set is lower than the optimal cartel price for this market condition. However, when information is shared with the manufacturer, such a situation cannot happen; the manufacturer's wholesale price signals to all the retailers that at least one retailer observed the informative signal. Consequently, all the cartel members set a high retail price.

The coordination problem of the cartel is severe under two cases. First, when there are many members in the cartel, which translates to a high probability that given high market demand, there is at least one cartel member who observes the non-informative signal. The second case is when the difference in the market potential between the two possible states of demand is high. In this case, the cartel suffers significant losses every time a coordination problem occurs. From the manufacturer's perspective, by agreeing to receive the shared information, she facilitates collusion at the retail level. However, the benefit to the manufacturer is the ability to set the wholesale price to better match the demand based on the retailers' signals. The following proposition highlights the importance of receiving this information from the manufacturer's perspective.

Proposition 7 *If $E[Var(A_i|\mathbf{Y}_n)] \geq \frac{A_\mu^2}{8}$, and $\psi \geq 3$, the manufacturer is better-off in setting $S3$ compared with setting $S2$.*

Proposition 7 asserts that if the market is characterized by high volatility, the manufacturer prefers to receive the shared information, even though by doing so she facilitates the formation of the cartel. When the manufacturer considers whether to accept the shared information from the retailers, she weighs the effect of increasing the scope of collusion as opposed to the benefit of improving her information about the market demand. When the market demand is volatile, the value of receiving the information from the retailers is high, since it enables the manufacturer to make better pricing decisions.

While the goal of the current paper is not to provide a detailed guideline to antitrust authorities with respect to the instances when vertical information-sharing facilitates collusion, Propositions 6 and 7 demonstrate the conditions under which the retailers would prefer to share their information with the manufacturer, and when the latter would agree to receive this information, knowing that she facilitates collusion. In particular, these propositions highlight two factors that affect the formation of the cartel using vertical information-sharing: the number of retailers in the market and the uncertainty level in the market. Proposition 6 shows that as the number of retailers in the market increases, the coordination problem becomes more severe; thus, the retailers, seeking for ways to mitigate this problem, can choose to share their information with the mutual manufacturer. Proposition 7 suggests that the manufacturer agrees to receive this information when the market is characterized by high uncertainty.

The next issue worth exploring is the effect that banning horizontal communication would have on consumer surplus. I focus on a comparison between setting $S1$—in which the retailers are allowed to exchange information horizontally—and setting $S3$, in which the retailers collude by sharing information with the manufacturer due to their inability to share information horizontally. The main reason behind the policy that limits horizontal information-exchange among competing firms is the risk that such practice would lead to collusion, which—in turn—would result in reduced consumer surplus. This paper suggests that as a result of the unavailability of the option to share information directly, the retailers would not forego the option of colluding, but can choose to obtain this desired result by sharing information with the manufacturer. The next proposition compares the effect on the consumer surplus when the retailers collude by sharing information with the manufacturer and when they can share information among the retailers.

Proposition 8 *Let CS^{S1} be the consumer surplus in setting $S1$, and CS^{S3} be the consumer surplus in* *setting* $S3$. *If* $\psi \geq 3$, *then*

$$CS^{S1} > CS^{S3}.$$

The proposition demonstrates that limiting the ability of the retailers to establish a cartel by sharing information horizontally can result in lower consumer surplus, which is generated when the retailers share information with the manufacturer. Thus, the current antitrust policy may have the perverse effect of not limiting the scope of collusion and lowering the consumer surplus. The intuition behind this result is that in both settings the retailers are able to collude, but in setting $S3$, they provide information about the market demand to their manufacturer. As a result, the manufacturer is able to set the wholesale price to better match demand, thus exacerbating the problem of double marginalization (Spengler 1950, Pasternack 1985) and hurting the consumers. This proposition suggests that limiting the ability of the retailers to collude by using horizontal information-sharing can act as a two-edged sword: it does not affect the ability of the retailers to collude, since the retailers can decide to collude by sharing information with the manufacturer, and it can result in lower consumer surplus.

8 Conclusions

In this paper, I examine retailers' ability to share information with a mutual manufacturer to achieve collusion without resorting to direct communications, which are heavily sanctioned by the antitrust law and policies. In particular, the retailers share their private information with the manufacturer on the expected market demand and use the wholesale price to determine the collusive supra-competitive price. I show that, in certain instances, establishing a cartel based on the information flow from the retailers to their mutual manufacturer results in higher profit to the retailers than from any other traditional cartel formation mechanism. Moreover, often even a manufacturer who is cognizant of the fact that the retailers exploit her wholesale price to coordinate on a monopoly-pricing scheme would find it beneficial to accept the retailers' private information.

Information-sharing has received considerable attention in the literature on channel coordination. Recent research has shown that in a complex environment—such as one with multiple competing retailers and one mutual manufacturer—firms might be reluctant to share information, because their private information can be leaked to unintended recipients. Contrary to this stream of literature, I demonstrate that retailers may use the fact that their shared information can reach a third party to establish the cartel. Each retailer shares his private information with the manufacturer, anticipating that this information will be leaked to the competing retailers via the wholesale price. In this paper, I highlight the positive effect of 'information leakage' from the retailers' perspective.

In addition to the theoretical interest, the results presented in this research have important implications from the perspective of antitrust policy and enforcement mechanisms. In the absence of antitrust regulations, rational, patient, profit-maximizing firms would freely act in consort to reach a consensus and to maintain prices above the competitive level. However, in light of potential exposure to antitrust investigations and as a result of the risk of bearing sanctions, competing firms devise measures to achieve supra-competitive prices, while escaping authorities' scrutiny. Vertical information-sharing in a distribution channel, such as that analyzed in this work, is one strategy by which a group of retailers, endowed with private information about market demand, can achieve collusive outcomes. In fact, in this case, the retailers exchange information and achieve collusion while eluding the scrutiny of the antitrust enforcement, which currently fails to appreciate that the vertical sharing of information in a distribution channel may generate collusive outcomes.

This work suggests that the antitrust enforcement policy, which currently views information-sharing between competing firms as a practice that can lead to an inference of illegal collusion, should be updated to include some forms of vertical information-sharing in a distribution channel as supplementary evidence for possible collusive behavior. The paper also highlights a few factors that may indicate when vertical information-sharing can be viewed as a signal of collusion. In particular, as the number of cartel members or market volatility increases, it is more likely that the colluding retailers and the manufacturer will engage in vertical information-sharing.

Appendix

Proof of Lemma 1. (a) I first show that when $\delta \geq \overline{\delta}$ the retailers are able to set the monopoly price for any given wholesale price w. In the extreme case the manufacturer sets the wholesale price w so high that the retailers do not sell during periods in which they observe non-informative signals. In this case, the IC constraint during periods of high demand is

$$\pi(A_H, p_H^m, w) + \frac{\delta}{1-\delta}\pi(A_H, p_H^m, w)\mu(1-(1-\rho)^n) \geq \pi(A_H, p_H^m - \epsilon, w), \tag{18}$$

which can simplified to the expression

$$\delta \geq \frac{n-1}{n-(1-\mu(1-(1-\rho)^n)} = \overline{\delta}.$$

Note that if the retailers are able to sell during periods in which they receive non-informative signals, the LHS of (18) increases, and therefore the retailers are able to set the monopoly price during periods of high demand even for a lower discount factor.

If δ is high enough, the retailers set the monopoly price during both information sets. In this case $p_H = \frac{A_H + w}{2}$ and $p_\phi = \frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{2[\mu(1-\rho)^n + (1-\mu)]} + \frac{w}{2}$. The sold quantity, from the manufacturer's perspective is given by $E[Q] = \frac{\mu A_H + (1-\mu)A_L - w}{2}$, and the optimal wholesale price is given by $w = \frac{\mu A_H + (1-\mu)A_L}{2}$.

(b) When all the retailers share their private information with the manufacturer, she sets the wholesale price $w = \frac{A_H}{2}$ during the informative information set, and the wholesale price $w = \frac{\widetilde{A}_L}{2}$ in the non-informative information set. As a result, the ex-ante profit of the retailers is given by:

$$
\begin{aligned}
&\frac{\mu(1-(1-\rho)^n)}{16} A_H^2 + ((1-\mu)+\mu(1-\rho)^n) \frac{\widetilde{A}_L^2}{16} \\
&= \frac{\mu(1-(1-\rho)^n)}{16} \left(A_\mu + A_H - A_\mu\right)^2 + \frac{((1-\mu)+\mu(1-\rho)^n)}{16} \left(A_\mu + \widetilde{A}_L - A_\mu\right)^2 \\
&= \frac{\mu(1-(1-\rho)^n)}{16} \left(A_\mu^2 + \left(A_H - A_\mu\right)^2 + 2\left(A_\mu(A_H - A_\mu)\right)\right) + \frac{((1-\mu)+\mu(1-\rho)^n)}{16} \left(A_\mu^2 + \left(\widetilde{A}_L - A_\mu\right)^2 + 2A_\mu \left(\widetilde{A}_L -\right.\right. \\
&= \frac{A_\mu^2}{16} + \frac{E[Var(A_i|Y_n)]}{16} + \frac{1}{8} A_\mu \left[\underbrace{\mu(1-(1-\rho)^n) A_H + ((1-\mu)+\mu(1-\rho)^n) \widetilde{A}_L}_{=A_\mu} - A_\mu\right] \\
&= \frac{A_\mu^2}{16} + \frac{E[Var(A_i|Y_n)]}{16}
\end{aligned}
$$

When the retailers do not share their information with the manufacturer she sets the price during all periods to be $w = \frac{A_\mu}{2}$. In this case, the one-period ex-ante profit of the cartel is given by:

$$
\begin{aligned}
\Pi_i &= [\mu(1-\rho)^n + 1 - \mu] \left(\frac{\widetilde{A}_L}{2} - \frac{A_\mu}{4}\right)^2 + \mu(1-(1-\rho)^n) \left(\frac{A_H}{2} - \frac{A_\mu}{4}\right)^2 \\
&= \frac{A_\mu^2}{16} + [\mu(1-\rho)^n + 1 - \mu] \frac{\widetilde{A}_L^2}{4} + \mu(1-(1-\rho)^n) \frac{A_H^2}{4} - \frac{A_\mu^2}{4} \\
&= \frac{A_\mu^2}{16} + \frac{1}{4} E[Var(A_i|Y_n)],
\end{aligned}
$$

which is higher than the cartel's profit when information is shared with the manufacturer.

Proof of Proposition 1. (a) By Lemma 1 it was shown that the off-schedule constraints are slack for $\delta > \overline{\delta}$. Therefore, this lemma focuses only the on schedule constraint of a retailer observing the non-informative signal. When a retailer observing the non-informative signal sets the price p_H, there is a strictly positive probability that demand is actually low. In this case, all the retailers can infer that a retailer setting the high price has deviated from the collusive plan, and they initiate a price war. As a result, for a high enough discount factor such a deviation is not profitable.

(b) Assume the constraint given in Proposition 1 is relaxed. Then the cartel solves an unconstrained optimization problem. When observing the informative signal the cartel member knows that the demand is high, and when observing the non-informative signal, the cartel members updates his belief about the status of demand. The price p_H is implicitly given by $\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0$. If the cartel member observes the non-informative signal he sets the price p_ϕ which is implicitly given by $\mu(1-\rho)\frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi} + (1-\mu)\frac{\partial \pi(A_L, p_\phi, w)}{\partial p_\phi} = 0$ according to his belief system.

A retailer who observes an informative signal and sets the price p_H earns an expected profit of $\rho^{n-1}\pi(A_H, p_H, w)$. With probability ρ^{n-1} all other retailers observe the informative signal as well, and with probability $(1-\rho^{n-1})$ at least one retailer observes the non-informative signal. In the latter case, a retailer setting the price p_H earns zero profit. A retailer who decides to deviate and set the price p_ϕ earns the profit

$\sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_H^{n-1-k}, p_\phi^{k+1}, w)$. Therefore, the cartel can implement this solution, if a retailer does not find it beneficial to deviate. This is given by the condition:

$$\rho^{n-1}\pi(A_H, p_H, w) \geq \sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_H^{n-1-k}, p_\phi^{k+1}, w).$$

Proof of Proposition 2. When a retailer observes the informative signal his expected profit is given by

$$\rho^{n-1}\pi(A_H, p_H, w) + \delta V + (1-\rho^{n-1})\left[(1-\beta)\delta V + \beta\delta^{T+1}V\right]. \tag{19}$$

With probability ρ^{n-1} all other retailers observe the informative signal, and there is no price war. With probability $(1-\rho^{n-1})$ at least one retailer observes the non-informative signal, and hence the cooperation continues with probability $(1-\beta)$.

When a retailer observes the non-informative signal his expected profit is given by:

$$\frac{(1-\mu)}{1-\mu+\mu(1-\rho)}\pi(A_L, p_\phi, w) + \delta V + \frac{\mu(1-\rho)}{1-\mu+\mu(1-\rho)}\left[\sum_{k=0}^{n-1} P_k^{n-1}\pi(A_H, p_\phi^{k+1}, w) + (1-\beta)\delta V + \beta\delta^{T+1}V\right]. \tag{20}$$

In the first case, demand is low, and hence all retailers set the price p_ϕ. Since demand is low, cooperation continues in the next period. However with probability $\frac{\mu(1-\rho)}{1-\mu+\mu(1-\rho)}$ demand is high, and the retailer shares the market with all other retailers observing the non-informative signal. In this case, the cooperation continues only with probability $(1-\beta)$.

Using equations (19) and (20) we can solve for V :

$$\begin{aligned} V &= \mu\rho^n\left[\pi(A_H, p_H, w) + \delta V\right] + \mu(1-\rho^n)\left[\pi(A_H, p_\phi, w) + (1-\beta)\delta V + \beta\delta^{T+1}V\right] \\ &\quad + (1-\mu)\left[\pi(A_L, p_\phi, w) + \delta V\right] \\ V &= \frac{\mu\rho^n\pi(A_H, p_H, w) + \mu(1-\rho^n)\pi(A_H, p_\phi, w) + (1-\mu)\pi(A_L, p_\phi, w)}{1-\delta\left[\mu\rho^n + \mu(1-\rho^n)\left[(1-\beta)+\beta\delta^T\right] + (1-\mu)\right]} \end{aligned}$$

The constraint $IC-on-H$ is given by:

$$\begin{aligned} \rho^{n-1}\left[\pi(A_H, p_H, w) + \delta V\right] + (1-\rho^{n-1})\left[(1-\beta)\delta V + \beta\delta^{T+1}V\right] \geq &\sum_{k=0}^{n-1} P_k^{n-1}\pi(A_H, p_\phi^{k+1}, w) \\ &+ (1-\beta)\delta V + \beta\delta^{T+1}V. \end{aligned} \tag{21}$$

In this case, the cartel solves the following problem:

$$\max V$$

$$s.t.$$

Equation 21

The Lagrangian of the constrained problem is given by

$$\mathcal{L}(p_H, p_\phi, \beta, \lambda) = V + \lambda \left[\rho^{n-1}\pi(A_H, p_H, w) + \rho^{n-1}\beta V(\delta - \delta^T) - \sum_{k=0}^{n-1} P_k^{n-1}\pi(A_H, p_\phi^{k+1}, w) \right]$$

Solving for the FOC with respect to p_H gives that:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial p_H} = & \frac{\mu\rho^n}{1-\delta\left[\mu\rho^n + \mu(1-\rho^n)\left[(1-\beta)+\beta\delta^T\right] + (1-\mu)\right]} \frac{\partial \pi(A_H, p_H, w)}{\partial p_H} + \lambda\rho^{n-1}\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} \\ & + \lambda\rho^{n-1}\beta(\delta-\delta^T)\frac{\mu\rho^n}{1-\delta\left[\mu\rho^n + \mu(1-\rho^n)\left[(1-\beta)+\beta\delta^T\right] + (1-\mu)\right]} \frac{\partial \pi(A_H, p_H, w)}{\partial p_H}. \end{aligned}$$

Note that at the point $\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0$ the FOC is also equal to zero. This proves the first part of the proposition, that the cartel sets the price for a retailer observing the informative signal at the point where $\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0$.

The FOC with respect to p_ϕ gives that:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial p_\phi} = & (1 + \lambda\rho^{n-1}\beta\left(\delta - \delta^T\right)) \frac{\mu(1-\rho^n)\frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi} + (1-\mu)\frac{\partial \pi(A_L, p_\phi, w)}{\partial p_\phi}}{1-\delta\left[\mu\rho^n + \mu(1-\rho^n)\left[(1-\beta)+\beta\delta^T\right] + (1-\mu)\right]} \\ & - \lambda\sum_{k=0}^{n-1} P_k^{n-1}\frac{\partial \pi(A_H, p_\phi^{k+1}, w)}{\partial p_\phi}. \end{aligned}$$

Note that at the point p_ϕ^* (the price at the non-constrained problem) the first term is negative. The term $\frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi}|_{p_\phi = p_\phi^*} > 0$, and hence the second term is negative. The value of $\frac{\partial \mathcal{L}}{\partial p_\phi}|_{p_\phi = p_\phi^*} < 0$, and hence at the constrained problem, the cartel reduces the price during periods in which a retailer observes a non-informative signal.

Proof of Lemma 2. When the retailers choose a rigid pricing scheme, they choose the price p by solving the problem

$$\max_p \mu(A_H - p)(p - w) + (1-\mu)(A_L - p)(p - w).$$

The solution to this problem is $p = \frac{A_\mu + w}{2}$, and the expected sold quantity is $Q = \frac{A_\mu - w}{2}$. In this case, the manufacturer sets the wholesale price $w = \frac{A_\mu}{2}$, and the profits of the manufacturer and the retailers are as suggested by the Lemma.

Proof of Proposition 3. Denote the rigid price by p_μ. Consider the upper-bound on the cartel's profit under the separating pricing strategy. In this case, the solution (p_H^*, p_ϕ^*) is feasible. Note that $p_\phi^* < p_\mu < p_H^*$, and also that $\pi(A_H, p_\phi^*) < \pi(A_H, p_\mu)$. The ex-ante profit of the cartel under the rigid pricing is $\mu\pi(A_H, p_\mu) + (1-\mu)\pi(A_L, p_\mu)$, and the ex-ante profit of the cartel under the separating pricing is given by:

$$\mu\left[\rho^n\pi(A_H, p_H^*) + (1-\rho^n)\pi(A_H, p_\phi^*)\right] + (1-\mu)\pi(A_L, p_\phi^*).$$

Let Δ be the difference between the two strategies, such that

$$\Delta = \mu\pi(A_H, p_\mu) + (1-\mu)\pi(A_L, p_\mu) - \left(\mu\left[\rho^n \pi(A_H, p_H^*) + (1-\rho^n)\pi(A_H, p_\phi^*)\right] + (1-\mu)\pi(A_L, p_\phi^*)\right).$$

Note that $\frac{\partial \Delta}{\partial n} > 0$, and for a high n, $\Delta > 0$.

Proof of Proposition 4. First, note that a manufacturer receiving only non-informative signals does not have an incentive to set the price w_H since this deviation is detected by the retailers with non-zero probability. However, a manufacturer receiving an informative signal, might have an incentive to set the low price in order to induce the retailers to set a low retail price. A manufacturer observing the informative signal will choose to set the price w_H if the following condition is satisfied:

$$\frac{A_H^2}{8} \geq \left(A_H - \frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{2\left[1-\mu+\mu(1-\rho)^n\right]} - \frac{w_\phi}{2}\right) w_\phi. \tag{22}$$

The LHS denotes the profit of the manufacturer from setting the price w_H. The RHS denotes the manufacturer's profit from setting the price w_L. It is possible to confirm that if:

$$\psi = \frac{A_H}{\frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{[1-\mu+\mu(1-\rho)^n]}} \geq 3,$$

the set of wholesale prices $w_H = A_H \backslash 2$ and $w_\phi = \widetilde{A}_L \backslash 2$ satisfies the condition given in equation (22). However, if $\psi < 3$, the condition given in equation (22) is not satisfied. In the latter case, solving equation (22) for equality provides the solution

$$w_\phi = \frac{2A_H - \widetilde{A}_L - \sqrt{3A_H^2 - 4A_H\widetilde{A}_L + \left(\widetilde{A}_L\right)^2}}{2}. \tag{23}$$

Proof of Proposition 5. In scenario $S2$ the profit of the retailers, under the rigid pricing scheme, during each period is $\Pi_i^{S2} = \frac{A_\mu^2}{16}$. When they share information with the manufacturer their expected profit during each period is at least (if the manufacturer lowers the price during the periods with non-informative signals, the profit of the retailers is higher)

$$\Pi_i^{S3} = \mu\left[(1-(1-\rho)^n)\frac{A_H^2}{16}\right] + \frac{(\mu(1-\rho)^n + (1-\mu))\left[A_H\mu(1-\rho)^n + (1-\mu)A_L\right]^2}{16}.$$

It is possible to show that at the point $\rho = 0$, $\Pi_i^{S3} = \Pi_i^{S2}$, and that Π_i^{S3} is increasing in $\rho \in (0,1)$. Therefore, the retailers prefer to share information with the manufacturer over the rigid pricing scheme.

The manufacturer's profit in scenario $S2$ under the rigid pricing scheme is $\Pi_M^{S2} = \frac{A_\mu^2}{8}$, and in scenario $S3$ it is

$$\Pi_M^{S3} = \mu\left[(1-(1-\rho)^n)\frac{A_H^2}{8}\right] + \frac{(\mu(1-\rho)^n + (1-\mu))\left[A_H\mu(1-\rho)^n + (1-\mu)A_L\right]^2}{8},$$

if $\psi > 3$. When $\rho = 0$, $\Pi_M^{S3} = \Pi_M^{S2}$, and that Π_M^{S3} is increasing in $\rho \in (0,1)$. Therefore, in this case the manufacturer is also better-off accepting information from the retailers.

Proof of Proposition 6. The ex-ante per-period profit of the cartel in scenario $S2$, under the separating pricing, is given by:

$$\Pi^{S2} = (1-\mu)\pi(A_L, p_\phi^*, w_\mu) + \mu(1-\rho^n)\pi(A_H, p_\phi^*, w_\mu) + \mu\rho^n\pi(A_H, p_H^*, w_\mu).$$

The ex-ante per-period profit of the cartel in scenario $S3$ is given by:

$$\Pi^{S3} = (1-\mu)\pi(A_L, p_\phi^{S3}, w_L^{S3}) + \mu(1-\rho)^n\pi(A_H, p_\phi^{S3}, w_L^{S3}) + \mu(1-(1-\rho)^n)\pi(A_H, p_H^{S3}, w_H^{S3}).$$

Note that $\pi(A_L, p_\phi^{S3}, w_L^{S3}) > \pi(A_L, p_\phi^*, w_\mu)$ and that Π^{S2} is decreasing in n, while Π^{S3} is increasing in n. Therefore for n high enough, $\Pi^{S3} > \Pi^{S2}$.

Proof of Proposition 7. Consider first the extreme case, in which the upon refusing to receive the retailers' information, the cartel collapses and the retailers compete by pricing the product at their marginal cost of w. In this case, the expected profit of the manufacturer is $\frac{A\mu^2}{4}$. In setting $S3$, when $\psi \geq 3$, the expected profit of the manufacturer is given by:

$$\begin{aligned}
\Pi_M^{S3} &= \mu(1-(1-\rho)^n)\frac{(A_H)^2}{8} + (\mu(1-\rho)^n + (1-\mu))\frac{\left(\widetilde{A}_L\right)^2}{8} \\
&= \mu(1-(1-\rho)^n)\frac{(A_H + A_\mu - A_\mu)^2}{8} + (\mu(1-\rho)^n + (1-\mu))\frac{\left(A_\mu - A_\mu + \widetilde{A}_L\right)^2}{8} \\
&= \frac{(A_\mu)^2}{8} + \frac{\mu(1-(1-\rho)^n)}{8}\left[(A_H - A_\mu)^2 + 2(A_H - A_\mu)A_\mu\right] \\
&\quad + \frac{(\mu(1-\rho)^n + (1-\mu))}{8}\left[\left(\widetilde{A}_L - A_\mu\right)^2 + 2\left(\widetilde{A}_L - A_\mu\right)A_\mu\right] \\
&= \frac{(A_\mu)^2}{8} + \frac{E[Var(A_i|Y_n)]}{8},
\end{aligned}$$

since $\frac{\mu(1-(1-\rho)^n)}{8}2(A_H - A_\mu)A_\mu + \frac{(\mu(1-\rho)^n+(1-\mu))}{8}2\left(\widetilde{A}_L - A_\mu\right)A_\mu = 0$.

Proof of Proposition 8. Since the demand function is linear, the consumer surplus is $\frac{1}{2}Q^2$ and can be expressed as:

$$CS^{S3} = \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\frac{A_H}{4}\right)^2 + \mu(1-\rho)^n\left[A_H - \frac{3}{4}\widetilde{A_L}\right]^2 + (1-\mu)\left[A_L - \frac{3}{4}\widetilde{A_L}\right]^2\right], \tag{24}$$

and

$$CS^{S1} = \frac{1}{2}\left[\begin{array}{c}\mu(1-(1-\rho)^n)\left(\frac{A_H}{2}-\frac{A_\mu}{4}\right)^2+\mu(1-\rho)^n\left[A_H-\frac{1}{2}\widetilde{A_L}-\frac{A_\mu}{4}\right]^2\\ +(1-\mu)\left[A_L-\frac{1}{2}\widetilde{A_L}-\frac{A_\mu}{4}\right]^2\end{array}\right] \tag{25}$$

$$= \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\frac{A_H}{4}+\frac{A_H-A_\mu}{4}\right)^2+\mu(1-\rho)^n\left[A_H-\frac{3}{4}\widetilde{A_L}+\frac{\widetilde{A_L}-A_\mu}{4}\right]^2\right]$$

$$+\frac{1}{2}\left[(1-\mu)\left[A_L-\frac{3}{4}\widetilde{A_L}+\frac{\widetilde{A_L}-A_\mu}{4}\right]^2\right].$$

Subtracting (24) from (25) gives that:

$$CS^{S1}-CS^{S3} = \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\underbrace{\left(\frac{A_H-A_\mu}{4}\right)^2}_{*}+\underbrace{A_H(\frac{A_H-A_\mu}{8})}_{**}\right)\right]$$

$$+\frac{1}{2}\left[(\mu(1-\rho)^n+(1-\mu))\underbrace{\left(\frac{\widetilde{A_L}-A_\mu}{4}\right)^2}_{*}\right]$$

$$+\underbrace{\frac{1}{2}\left(\frac{\widetilde{A_L}-A_\mu}{2}\right)\left(\frac{\widetilde{A_L}}{4}\right)}_{**}(\mu(1-\rho)^n+(1-\mu))$$

Note that summation of the two terms marked by $*$ results in $E[Var(A_{i|}Y_n)]$, and the summation of the two terms marked by $**$ results also in $E[Var(A_{i|}Y_n)]$. Therefore the expression $CS^{S1}-CS^{S3}>0$.

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